Financial Constraints, Competition and Hedging in Industry Equilibrium∗

Abstract
We show that, under standard assumptions about technology and production costs, financially constrained firms may have incentives to hedge as well as to speculate. Whether a firm increases or decreases its risk exposure depends on the hedging and investment strategies of its competitors. We show that in equilibrium even identical firms may choose different hedging strategies. Industry characteristics, such as the degree of competition, the number of firms in the industry, the size of the market, the elasticity of demand, and marginal production costs determine how many firms hedge and how many firms do not hedge in equilibrium. Even if hedging strategies are continuous, the typical equilibria are corner solutions, in which some firms remain completely unhedged and others hedge to the maximum possible extent. Our results explain several stylized facts documented in surveys and recent empirical work, and generate new testable hypotheses concerning the determinants of hedging strategies in an industry context.

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1 Introduction

There is now substantial evidence that firms behave as if capital raised externally, through debt or equity offerings, is more costly than internally generated funds. An implication of this observation, originally posited by Lessard (1990), and formalized and extended by Froot, Scharfstein and Stein (FSS) (1993), is that firms can improve the expected productivity of their investment expenditures by increasing the correlation between internally generated funds and the marginal productivity of their capital. In other words, firms should manage their risk exposures so that cash flows are generated in states of the economy in which investments generate the highest returns. Within the context of the FSS model this means that firms will hedge their cash flows if the following three conditions hold: (i) there is a cost to raising external funds, which is convex in the amount raised, (ii) the correlation between unhedged cash flows and investment opportunities is not too high, and (iii) the profit function is concave in the level of investment.

Although there is empirical evidence consistent with the idea that financial constraints play an important role in risk management decisions,1 recent empirical studies and surveys provide a number of stylized facts about corporate hedging practices that have not been adequately addressed. Broadly, three sets of issues have emerged. First, in most studies approximately half of the sample firms actively hedge, while the other half does not.2

1 For example, Geczy et al. (1997) find that firms with tighter financial constraints and more growth opportunities are more likely to use currency derivatives.

Second, the cross-sectional correlation between firm-specific characteristics, suggested by theory, and hedging practices is relatively weak. Third, there is considerable variation across industries in the extent to which firms hedge, and there is somewhat more limited evidence that industry characteristics such as the degree of competition affect the decision and the extent of hedging in an industry.

To accommodate these observations, our analysis extends the FSS model in two important ways. First, we weaken the FSS assumption that the profit function is always concave in investment. Our model considers a standard textbook setting in which the long-run average cost curve is U-shaped. In this setting, the expected production cost for any given level of output is higher if investment is random as opposed to being at its expected value. However, a firm can potentially benefit from random investment in this setting since it chooses the level of production after learning the realization of its production costs, i.e., it produces more when its production costs are lower, and less when costs are higher. If the firm is financially unconstrained, then second-order conditions imply that the profit function is locally concave at the optimal level of investment. However, if financial constraints are binding, and a firm’s...
investment is determined by the availability of internal funds, it can operate in the convex region of the profit function. When this is the case the firm can potentially benefit from an increase in the variability of its cash flows.5

Second, we consider firms’ risk management choices within the context of an industry equilibrium in which output prices are determined endogenously. Within this setting, a firm’s risk management choice is affected by the investment/hedging decisions of other firms.6 In our model, a firm’s marginal productivity of capital is higher when the aggregate level of capital is low. This implies that a firm has an incentive to make risk management choices that result in cash flows that are high when the aggregate level of investment is low and vice versa, allowing the firm to increase its own investment when its competitors are forced to cut back. This in turn implies that the incentive for an individual firm to hedge increases as more firms in the industry choose not to hedge and vice versa. As a result, an industry equilibrium can exist in which some of the ex-ante identical firms will hedge while others will not.

By assuming that firms either hedge completely or not at all, we obtain a closed-form expression for the fraction of firms that hedge in equilibrium. This closed-form solution allows us to examine how the fraction of firms that hedge in equilibrium depends on various industry characteristics.7 There is a limited amount of empirical evidence on how industry

5 An alternative way to think about this is that firms posses real options. It is well known that the value of options increase in the volatility of the underlying. Therefore firms pursue (hedging) strategies, that maximize the value of their real options, subject to certain constraints. In this context, risk is an opportunity for the firm to benefit from its real options.

6 Recent empirical evidence shows that the fraction of firms that hedge in an industry affects the volatility of the industry price. Nain (2004) finds that industry prices are less sensitive to foreign exchange rate movements in industries where currency hedging is more common. This is what our model predicts.

7 We show that our results extend to a setting in which firms can hedge partially, i.e., the hedging strategy
characteristics are related to the proportion of firms that hedge in an industry. For example, Allayannis and Weston (1999) find that in industries in which less than one-half of the firms hedge, industry mark-ups (which proxy for the competitiveness of the market) are negatively correlated with the proportion of firms that hedge. Geczy et al. (1997) find that in industries with many firms, there is more heterogeneity in hedging decisions than in industries with relatively few firms. Our results are consistent with these findings, and suggests additional testable hypothesis concerning the relation between industry characteristics and the heterogeneity of hedging practices within an industry. In particular, our model implies that in industries with more competitors, a steeper industry demand curve, and a flatter marginal cost curve, there is more heterogeneity in firms’ hedging choices, i.e., the fraction of firms that hedge in industry equilibrium moves towards 50% (the point of maximum heterogeneity). In addition, we find that larger market size causes a larger fraction of firms to hedge in equilibrium.

The model developed in this paper is related to existing research that explores why similar firms in the same industries often choose different capital structures. For example, our equilibrium analysis is similar to Maksimovic and Zechner (1991), who show that ex-ante identical firms may choose different debt-equity ratios. Both our paper and Maksimovic and Zechner (1991) are related to De Meza (1986), who shows that otherwise identical firms may choose different production technologies in an industry equilibrium. Our model is also closely related to Shleifer and Vishny (1992), who examine how a firm’s bankruptcy/liquidation costs depend on the financial health of its competitors. In the Shleifer and Vishny model capital structure choices within an industry are interdependent because the bankruptcy costs of an is represented by a continuous variable on the unit interval.
individual firm are lower when other firms within the same industry are financially healthy
and compete to buy the assets of failing firms. Similar to Shleifer and Vishny (1992), the
firms in our model have an incentive to generate cash flows in those situations in which other
industry participants are financially constrained.8

The rest of the paper is organized as follows. In Section 2, we outline the basic model and
motivate a production technology that exhibits first increasing and then decreasing returns-
to-scale. We show that the profit function associated with this production technology is
first convex and then concave in investment. In Section 3, we derive subgame-perfect Nash
equilibria (SPNE) for our two-stage game. In section 4, we derive the proportion of firms
that hedge in industry equilibrium, and establish comparative static results that relate the
proportion of firms that hedge to industry characteristics. Section 5 extends the analysis to
the case of continuous hedging strategies, and Section 6 concludes.

2 The Model

Consider an industry with $n$ identical firms. At date 1, all firms receive identical but uncertain
cash flows, which they can invest in productive capital that generates returns at date 2. One
can think of these cash flows as the amount of cash denominated in foreign currency generated
from past production decisions. The foreign currency risk causes the value of these cash flows
to be uncertain.9

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8 In a recent paper, Mello and Ruckes (2004) also consider how hedging decisions of firms competing in
the same product market can be interdependent. They assume a convex payoff function, which implies that
the benefit from "getting ahead" is larger than the loss from "falling behind" in product market competition.
This convexity also leads to heterogeneity in firms' hedging decisions.

9 Alternatively, one could assume that firms have exposures after production decisions are made. For
example, they may have accounts receivable or payable that are affected by exchange rate movements, or they
may have entered into forward contracts with foreign suppliers or buyers at prices that are fixed in terms of
At date 0, firms decide whether or not to hedge cash flows. If a firm does not hedge its date 1 cash flows, the amount of funds available for investment at date 1 will be \( y = \bar{y} + \epsilon \), where \( \epsilon \) is a common shock to all firms. We denote \( E(y) = \bar{y} \). If a firm hedges, then the amount of funds available for investment at date 1 will be \( y = \hat{y} \).\(^{10}\)

We assume that firms are financially constrained, i.e., for all values of \( y \), the marginal return to investment is higher than the opportunity cost of funds, which is normalized to 1. This implies that firms will invest their entire cash flows.\(^{11}\) Therefore, \( y = k \), where \( k \) denotes a firm’s investment. At date 1, firms also supply additional factor inputs, such as raw materials and labor, to produce a quantity of goods that are sold at date 2. The above discussion is summarized in the following time line of events:

- **T=0** Firms only know the exogenous distribution of their cash endowments \( y \) at \( T=1 \), not the realization. Each firm decides whether to completely hedge its cash flow risk or to remain completely unhedged. Each firm chooses its optimal hedging strategy given the hedging strategies of all other firms. That is, hedging is determined within a Nash equilibrium.

- **T=1** Cash flows are realized, and firms invest all of their cash flows. Each firm observes the investment decisions of all other firms. Firms then choose their outputs in the context of a Cournot equilibrium.

- **T=2** Profits are realized.\(^{12}\)

\(^{10}\) In this section we consider only two possibilities: the firm can either hedge completely or remain completely unhedged. We consider partial hedging strategies in a later section.

\(^{11}\) Alternatively, empire-building tendencies also may motivate managers to invest all of their cash flows.

\(^{12}\) Mello and Ruckes (2004) present a model in which exchange rate movements affect firms’ payoffs depending on the currency denomination in which debt is issued.
2.1 Technology, costs and profits

The production decision at T=1 is based on a production technology that requires two variable inputs, raw materials and labor, whose quantities are denoted by $X_1$ and $X_2$, and a fixed input, capital, whose quantity is denoted by $k$. At T=1, $k$ is predetermined, and thus independent of the choices of $X_1$ and $X_2$. The production function is assumed to be of the following form:

$$q = \min[\gamma(k)X_1, \frac{1}{2}X_2^2], \quad (1)$$

where $\gamma(k)$ is an increasing and weakly concave function, i.e., $\frac{d\gamma}{dk} > 0$ and $\frac{d^2\gamma}{dk^2} \leq 0$. We chose this specification of the production function so that the associated cost function is convex in output, and an increase in the capital stock $k$ reduces both variable and marginal production costs. In particular, a higher $k$ decreases the requirement for the variable factor input $X_1$ per unit of output, i.e., it increases the productivity of $X_1$.

The production function implies that producing $q$ units of output requires at least $\frac{1}{\gamma(k)}q$ units of $X_1$ and $q^2$ units of $X_2$. The variable cost function associated with this production function is therefore

$$C(q) = \frac{\delta}{2}q^2 + \frac{c}{\gamma(k)}q, \quad (2)$$

where $\frac{\delta}{2}$ can be thought of as the cost of the variable factor input $X_2$ per unit of output, and $c$ as the cost of the variable factor input $X_1$ per unit of output. We show in Appendix A that this is a conventional cost function with a U-shaped long-run average cost curve. The

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12 As will be apparent below, assuming two variable inputs allows us to work in terms of a marginal production cost function with the properties that (i) it is increasing in output and (ii) capital investment affects only the intercept, and not the slope, of the marginal cost curve. Although we assume Cournot competition for the second stage, the case of price-taking firms is a useful benchmark. The former property is required if we are to accommodate this special case into our analysis, and the latter is necessary for analytical tractability.
cost function is convex in output $q$ for a given level of $k$, i.e., the marginal cost is increasing in $q$. Note that $\delta$ measures the convexity of the cost function with respect to $q$. Given our assumptions about the function $\gamma(k)$, the cost function is convex in $k$ for a given level of output, $q$. As we shall see below, this is relevant for the firm’s T=0 decision.

2.1.1 The profit function for a price-taking firm

To clarify our intuition, it is useful to first analyze the characteristics of the profit function of a price-taking firm, i.e., a firm that ignores the effect of its own output decision on the equilibrium output price. We start at T=1 with a single firm facing a given output price $P$.

For a given $k$, profit maximization involves setting $q$ such that

$$P = C_q = \delta q + \frac{c}{\gamma(k)}. \tag{3}$$

Hence, the firm’s profit function is given by

$$\Pi(k) = Pq - C(q,k) = Pq - \left( \frac{\delta}{2} q^2 + \frac{c}{\gamma(k)} q \right) = \frac{\left( P - \frac{c}{\gamma(k)} \right)^2}{2\delta}. \tag{4}$$

Production takes place only if $P - \frac{c}{\gamma(k)} > 0$. Therefore the profit function can be written as

$$\Pi(k) = \frac{1}{2\delta} \max \left\{ \left( P - \frac{c}{\gamma(k)} \right), 0 \right\}^2. \tag{5}$$

Before considering the curvature of the profit function with respect to $k$, which determines a firm’s incentive to hedge, note that for a given $k$, the profit function is convex in $P$. This convexity arises because the firm has the option to increase or decrease $q$ after observing $P$. We refer to this option as production flexibility. The firm can adjust its production subject
to current market conditions. The greater the sensitivity of \( q \) to \( P \), i.e., the larger is \( \frac{\partial q}{\partial P} \), the greater is the convexity of the profit function. This can be checked easily by noting that

\[
\frac{\partial^2 \Pi}{\partial P^2} = \frac{\partial q}{\partial P} = \frac{1}{\delta}.
\]

(6)

Thus, if \( k \) is fixed the convexity of the profit function in \( P \) implies that the firm prefers \( P \) to have a higher variance. This observation will be useful later in understanding the firm’s incentives to hedge, when we allow the market price to be affected by the investment choices of other firms. Notice also that the sensitivity of \( q \) to \( P \) is inversely proportional to \( \delta \). That is, the flatter the marginal cost curve, the greater is the production flexibility. In other words, the convexity of the cost function reduces the value of the firm’s production flexibility.

The curvature of the profit function with respect to \( k \) (which determines the price-taking firm’s incentive to hedge) can be evaluated by taking the second derivative of the profit function with respect to \( k \).

\[
\frac{\partial^2 \Pi(k)}{\partial k^2} = \frac{2c}{\delta k(\gamma(k))^2} \left[ \frac{\gamma''(k)k}{\gamma'(k)} - \frac{2\gamma'(k)k}{\gamma(k)} + \frac{ck}{P - \frac{c}{\gamma(k)}} \cdot \frac{\gamma'(k)}{\gamma(k)^2} \right].
\]

(7)

Notice that the first two terms within the brackets are negative, while only the third term is positive. This implies that the source of convexity must come from the third term, which relates to the firm’s ability to choose \( q \) after observing \( k \). Note that the elasticity of output with respect to \( k \) is given by

\[
\frac{\partial q}{\partial k} \cdot \frac{k}{q} = \frac{ck}{P - \frac{c}{\gamma(k)}} \cdot \frac{\gamma'(k)}{\gamma(k)^2},
\]

(8)

which is the third term in the bracketed expression. The greater the sensitivity of \( q \) to \( k \), the greater is the convexity of the profit function.
While the profit function is convex in $P$, it is convex in $k$ only for sufficiently small values of $P$. To see this, consider the case of $\gamma(k) = k^\phi$, where $0 < \phi \leq 1$. Then the bracketed expression in (7) reduces to

$$
(\phi - 1) - 2\phi + \frac{c\phi}{k^\phi P - c}
$$

(9)

Whether or not the profit function is locally convex or concave in $k$ at a particular value of $k$ (i.e., whether or not the second derivative of the profit function given in equation (7) is positive or negative) depends on the level of the price, $P$. If $P$ is sufficiently small so that the term $k^\phi P - c$ is small, then the third term, representing the elasticity of output with respect to $k$, dominates the remaining two negative terms. The opposite holds for larger values of $P$. Thus, the profit function is locally convex in $k$ for small values of $P$, and locally concave in $k$ for larger values of $P$.

The reason why the level of $P$ affects the curvature of the profit function is as follows. Since the cost function is convex in $k$, randomness in $k$ increases the expected cost of production for a given level of output, $q$. An increase in the expected cost of production lowers expected profit more if the output being produced is high, i.e., if $P$ is high. On the other hand, a random $k$ allows the firm to adjust output depending on the realization of $k$, which affects its marginal cost. This is the flexibility option. Thus the firm prefers a fixed $k$ to a

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13 Note that $k^\phi P - c > 0$. Otherwise no output is produced.

14 This conclusion does not depend on the particular functional form for $\gamma(k)$ assumed. Note that $P - \frac{c}{k^\phi} > 0$ for positive output to be produced. Now as $P$ approaches $\frac{c}{k^\phi}$ from above, the third term within square brackets in the right hand side of equation (7) becomes arbitrarily large, while all other terms remain bounded. Hence, the expression within square brackets becomes positive.

15 Notice that for given $P$, it is also the case that the profit function is convex in $k$ for low values of $k$ and concave for in $k$ for high values of $k$. At the unconstrained optimal level of $k$, however, by virtue of second order necessary conditions for a maximum, the profit function must be concave in $k$. Thus, financially constrained firms can operate in the convex region of their profit function if their investment levels are substantially below the unconstrained optimum.
random $k$ - i.e., the profit function is concave in $k$ - when $P$ is high. On the other hand, if $P$ is low, the value of the flexibility option dominates: the firm prefers $k$ to be random, i.e., the profit function is convex in $k$.

To summarize, a constrained firm benefits from two sources of uncertainty: (i) uncertainty about the output price, and (ii) uncertainty about its cash flow (capital stock $k$). The reason uncertainty is valuable is because the firm can choose its own output $q$ after the output price is realized and its production costs are determined. As we have shown, because of this production flexibility, uncertainty can be viewed as an opportunity rather than as a cost. As we show later, for an industry facing a negatively sloped demand curve, output prices at $T=2$ and cash flows at $T=1$ are negatively correlated in equilibrium. Thus, the two risk exposures tend to offset each other, which has a material effect on an individual firm’s decision to hedge its cash flow.

We now proceed to complete our specifications of the cost and demand conditions and the competitive environment in the product market. For the rest of the paper, we shall drop the assumption of price-taking firms.16

2.2 Hedging and production costs

For analytical simplicity, we shall assume for now that firms can either completely hedge their cash flows, or remain completely unhedged. In Section 5, we shall relax this assumption, and

16 Our analysis appears based on particular functional forms for production and cost. As the discussion in this section has shown, however, the crucial aspect of our analysis is that the profit function be convex for small $k$, which is associated with a region of increasing returns to scale (and a region of decreasing long-run average cost). These are exactly the types of production and cost functions that have been traditionally assumed in economic analysis. Thus, so long as these features are preserved under more general set ups, our results should go through. One of the benefits of assuming specific functional forms is that we are able to get closed-form solutions for some endogenous variables of interest, which makes the analysis much more tractable.
show that the flavor of the main results remains.

Recall from Section 2 that if a firm hedges, its cash flow at T=1 will be \( y = \bar{y} \), while if the firm does not hedge, its cash flow at T=1 will be \( y = \bar{y} + \epsilon \). Since firms invest their entire cash flows by assumption, we can use \( y \) and \( k \) interchangeably. Thus, \( k \) denotes the (random) investment of a firm that does not hedge, and \( \bar{k} \) denotes the (deterministic) investment of a firm that hedges.

Based on Equation (2) the cost function of a firm that hedges is given by

\[
C^h(q) = \frac{\delta}{2} q^2 + \frac{c}{\gamma(k)} q = \frac{\delta}{2} q^2 + \alpha q, \tag{10}
\]

where

\[
\alpha \equiv \frac{c}{\gamma(k)}. \tag{11}
\]

The cost function of a firm that does not hedge is given by

\[
C^u(q) = \frac{\delta}{2} q^2 + c \left( \frac{1}{\gamma(k)} - \frac{1}{\gamma(\bar{k})} \right) q + \frac{c}{\gamma(\bar{k})} q = \frac{\delta}{2} q^2 + wq + \alpha q, \tag{12}
\]

where

\[
\alpha = \frac{c}{\gamma(k)}
\]

and

\[
w \equiv c \left( \frac{1}{\gamma(k)} - \frac{1}{\gamma(\bar{k})} \right) \tag{13}
\]

is a random variable. Notice that our assumptions about \( \gamma \) imply that the function \( \frac{1}{\gamma} \) is convex. Thus, by Jensen’s Inequality,

\[
E(w) = cE \left( \frac{1}{\gamma(k)} - \frac{1}{\gamma(\bar{k})} \right) > 0. \tag{14}
\]
The random variable $w$ captures the effect of the firm’s cash flows (and investment, $k$) on its production cost. The fact that $E(w)$ is greater than zero implies that there is a cost advantage of hedging. The expected total (as well as average and marginal) production costs are lower for a hedged firm than for an unhedged firm. The reason is that the cost function is convex in $k$. Thus, reducing the volatility in $k$ reduces the firm’s expected production cost. However, as will be apparent below, the randomness in $k$ (or $w$) also gives the firm the flexibility to adjust production according to its realized production cost. This is the benefit of remaining unhedged.

2.3 Demand

The industry demand curve is assumed to be linear:

$$Q = a - \frac{P}{b},$$

where $a > 0$ and $b > 0$ are constants, and $Q$ denotes industry output.

2.4 Competition

We assume that firms are Cournot competitors in the product market. Each firm observes the hedging choices of the other firms as well as the realization of the common cash flow shock before deciding its output. Equilibrium requires each firm’s output choice to be a best response to the output choices of all other firms.

3 Subgame-Perfect Nash Equilibria (SPNE)

Our model is an example of a two-stage (dynamic) game, with the following structure:

*Stage 1:* All firms simultaneously decide whether to "hedge" or "not hedge". Hedging
implies that the firm’s cash flow and investment will be \( y = \bar{y} \) (i.e., constant), and not hedging implies that its cash flow and investment will be \( y = \bar{y} + \epsilon \) (i.e., random).

\textit{Stage 2:} After investing their cash flows and observing the investments made by other firms, all firms play a Cournot game in output.

Notice that the stage 2 subgames are standard Cournot games in which the firms’ cost functions are as given in section (2.2). The equilibrium outputs and product price in the second stage game will depend on the number of firms that chose not to hedge in stage 1 \((m^u)\) and the realization of the random variable \(w\), which represents the cost shock to an unhedged firm. Let \(\Pi^u(w, m^u)\) denote the profit of an unhedged firm in the second stage game, and \(\Pi^h(w, m^u)\) denote the profit for a hedged firm, and let \(E\Pi^u(w, m^u)\) and \(E\Pi^h(w, m^u)\) denote the corresponding expected profits before the uncertainty \(w\) has been realized.

In a pure strategy subgame perfect equilibrium (SPNE) of this model, no firm must want to change its hedging decision (i.e. its decision on whether to hedge or not hedge) given the hedging decisions of the other firms. There is an equilibrium with \(m^u^*\) firms choosing to remain unhedged if and only if

\[
E\Pi^h(w, m^u^*) > E\Pi^u(w, m^u^* + 1)
\]

and

\[
E\Pi^u(w, m^u^*) > E\Pi^h(w, m^u^* - 1).
\]

14
3.1 The Second Stage

We begin by considering the output decision of an unhedged firm after the cost shock \( w \) is realized. Combining Equations (4), (12), and (15) we obtain the profit function of an unhedged firm:

\[
\Pi^u(w, m^u, q_i) = Pq_i - \frac{\delta}{2}q_i^2 - wq_i - \alpha q_i
\]  
(16)

where \( P \) denotes the industry price. The first-order condition for a maximum is

\[
\frac{d\Pi^u}{dq_i} = P + q_iP\frac{\partial Q}{\partial q_i} - \delta q_i - w - \alpha = 0.
\]  
(17)

In a Cournot-Nash equilibrium, each firm’s output decision must maximize profit, given the output decisions of all other firms. Thus, \( \frac{\partial Q}{\partial q_i} = 1 \).

Substituting \( P_Q = -b \), and setting the marginal profit equal to zero determines the profit-maximizing level of output of an unhedged firm:

\[
q^u = \frac{P(w, m^u) - w - \alpha}{\delta + b}.
\]  
(18)

Similarly, the profit-maximizing output of a hedged firm is

\[
q^h = \frac{P(w, m^u) - \alpha}{\delta + b}.
\]  
(19)

The stage two equilibrium is defined by equations (18), (19) and the following market-clearing condition:

\[
q^u m^u + q^h m^h = Q = \frac{a - P(w, m^u)}{b}.
\]  
(20)
Substituting the optimal output into the profit function (16) yields the stage two profit for the unhedged firm to be:

$$
\Pi^u(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} (P(w, m^u) - w - \alpha)^2.
$$

(21)

Equivalent substitutions for the hedged firm yield

$$
\Pi^h(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} (P(w, m^u) - \alpha)^2.
$$

(22)

### 3.2 The First Stage

To facilitate the discussion of the first stage, it is useful to compare the expected profit of a firm that hedges and a firm that chooses not to hedge. Taking expectations, and subtracting (22) from (21) yields the difference in expected profits between not hedging and hedging

$$
E\Pi^u(w, m^u) - E\Pi^h(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} [E(w^2) - 2E ((P(w, m^u) - \alpha)w)].
$$

(23)

Solving for the market clearing price $P(w, m^u)$ from equations (18),(19) and (20), we get:

**Lemma 1** Consider an industry with $n$ firms, of which $m^u$ firms do not hedge. Then the difference between the expected profits from remaining unhedged and hedging is given by:

$$
E\Pi^u(w, m^u) - E\Pi^h(w, m^u) = \frac{(\delta + 2b)}{2(\delta + b)^2} \left[ E(w^2) - \frac{2(a - \alpha)(\delta + b)}{nb + \delta + b} E(w) - \frac{2bm^u}{nb + \delta + b} E(w^2) \right].
$$

(24)

**Proof.** Please see Appendix B. ■

The following proposition gives necessary and sufficient conditions for equilibrium.

**Proposition 2** Let $\overline{m}^u$ be the real number (if it exists) in the interval $[0, n]$ for which the right hand side of (24) is zero, $m^u_1$ be the largest integer value of $m^u$ for which the right-hand-side of (24) is positive, and $m^u_2 = m^u_1 + 1$ be the smallest integer value for which it is negative.
1. If \( \overline{m}^u \) is an integer value\(^{17} \), then the equilibrium number of firms that remain unhedged is given by \( \overline{m}^u \).

2. If \( \overline{m}^u \) is not an integer, two cases arise:

   (a) If \( E\Pi^u(w, m^u_1) < E\Pi^h(w, m^u_1) \), then the equilibrium number of firms that remain unhedged is \( m^u_1 \).

   (b) If \( E\Pi^u(w, m^u_2) > E\Pi^h(w, m^u_2) \), then the equilibrium number of firms that remain unhedged is \( m^u_2 \).

Proof. Please see Appendix B. ■

3.3 Discussion

We now discuss the factors that affect the decisions of the firms to hedge or to remain unhedged. To begin with, note the convexity of the profit functions (21) and (22). The reason for this convexity is the production flexibility that we discussed previously, which implies that firms benefit from uncertainty. A firm that hedges benefits from uncertainty in output prices \( P(w) \), and the lower expected production costs. A firm that does not hedge benefits from uncertainty in \( P(w) - w \). Whether hedging or not hedging is more profitable depends on the properties of the equilibrium output price - in particular, its sensitivity to the shock \( w \) - and the cost savings attainable by hedging. For example, if the equilibrium price were constant, a firm’s incentive not to hedge would be captured by the randomness in \( w \) or the term \( E(w^2) \). Randomness in \( w \) increases the expected profits because the firm can choose the quantity to be produced after learning the realization of cost shock \( w \). On the other hand, not hedging implies higher production costs (recall that \( E(w) > 0 \)). Thus, firms

\(^{17} \overline{m}^u \) as defined does not exist if either (i) the right hand side of (24) is positive for all \( m^u \leq n \), or (ii) it is negative for \( m^u \geq 0 \). In the former case, all firms remain unhedged in equilibrium. In the latter, all firms hedge in equilibrium.
face a trade-off. If the industry demand curve were flat \((b = 0)\), so that \(P(w) = P\) for all \(w\), then this trade-off only depends on the relative magnitudes of \(E(w^2)\) and \(E(w)\). If

\[
P > \alpha + \frac{1}{2E(w)/E(w^2)}.
\]

then (23) is negative and all firms hedge. Intuitively, a sufficiently high output price implies that firms operate in the concave region of their profit functions (see Equation (7)). On the other hand, if the opposite inequality holds, all firms choose not to hedge.

The reason why all firms make identical hedging decisions when the industry price is a constant is as follows. When the industry price does not change, a firm’s incentive to hedge is not affected by the hedging decisions of other firms. Since firms are ex-ante identical, they make identical decisions. If the demand curve is not perfectly elastic, the decision of a firm to hedge affects other firms’ incentives to hedge as well, since it affects the firm’s own production and thus the industry price. If all firms face the same shock to \(w\), then firms that are not hedged will adjust their production according to the realization of \(w\), causing the industry price to be volatile and co-vary positively with \(w\). An increase in the volatility of the price will generally increase the incentive of a firm to hedge, because the two risk exposures, i.e., the output price and the firm’s cash flow which affects \(k\) and hence \(w\), partially offset each other. As will become clear in the next section, the volatility of the output price is a function of the number of firms that hedge in the industry. Thus, the hedging choices of other firms will affect the hedging choice of an individual firm. In this case we can get heterogeneity in hedging strategies, as we will show in Section 4.

\[18\] With a flat industry demand curve, the Cournot model reduces to a model of perfect competition in which one firm’s output has no effect on the industry price and hence the profits of other firms in the industry.
3.3.1 Common versus idiosyncratic shocks

In this section, we first analyze a firm’s incentive to hedge in an environment in which the firms face idiosyncratic shocks to their cash flows instead of common shocks. Lemma 1 assumes that all firms face identical cash flow shocks (perfect correlation across firms). If firms face independent but identically distributed cash flow shocks, such that $\text{Cov}(w_i, w_j) = 0$, then the difference in expected profits between a firm that does not hedge and a firm that hedges is given by

$$\frac{(\delta + 2b)}{2(\delta + b)^2} \left[ E(w_i^2) - \frac{2(a - \alpha)(\delta + b)}{nb + \delta + b} E(w_i) - \frac{2b(m^w - 1)}{nb + \delta + b} (E(w_i))^2 - \frac{2b}{nb + \delta + b} E(w_i^2) \right].$$

(26)

Below, we will compare this expression with the corresponding expression when firms face a common shock, given by equation (24). We first note that with idiosyncratic shocks, the incentive for firm $i$ to remain unhedged increases in the volatility of $w_i$. Intuitively, a firm can realize higher profits when its marginal costs are random rather than constant if it is able to vary its production so that it produces more when its costs are lower and less when its costs are higher. Since firms choose output after observing the marginal production cost, a firm that remains unhedged is able to take advantage of this production flexibility. Moreover, a more volatile $w_i$ presents an unhedged firm with a greater opportunity to benefit from production flexibility. Thus, as we see from expression (26), for $n \geq 2$, the difference in the profit of a firm that remains unhedged and a firm that hedges is increasing in $E(w_i^2)$. This is the benefit of remaining unhedged. On the other hand, as discussed above, there is a cost of remaining unhedged as well. Since $E(w_i) > 0$, the expected cost of production for any given level of output will be higher for the unhedged firm than for the hedged firm.
Expression (26) shows that the difference in expected profit between a firm that does not hedge and a firm that hedges is decreasing in $E(w_i)$.

Notice that the ratio $\theta = \frac{E(w)}{E(w^2)}$ captures, ceteris paribus, the relative importance of the cost and the benefit from not hedging. From equation (13) it is clear that this ratio itself is an *exogenous* parameter in the model, since it depends only on the production technology and the distribution of the cash flow shock.

With common shocks to cash flows, the case we focus on in this paper, the benefit of production flexibility is reduced. As long as there are some firms that are not hedged, the output price covaries positively with the cost shock. This means that a firm’s cost is low precisely when the industry price is low, and high when the industry price is high, which reduces the ability of the firm to benefit from adjusting output according to the realization of its marginal cost. We argued above that the term $E(w^2)$ is related to the benefit from production flexibility that arises because of a random marginal cost of production. A comparison of equations (24) and (26) shows that for $m^w > 1$, the difference in the expected profits between an unhedged and a hedged firm increases less for any given increase in $E(w^2)$ when firms face common shocks rather than idiosyncratic shocks.

Moreover, for both cases, as the number of firms that remain unhedged increases, the difference in the expected profit of a firm that remains unhedged and a firm that hedges is reduced. For the case of idiosyncratic shocks, this happens because an increase in the number of firms that remain unhedged increases the expected industry price, since unhedged firms have a higher cost of production on average ($E(w_i) > 0$). This makes it more costly for any individual firm to choose not to hedge, since the alternative of hedging and having a lower
expected cost of production is more attractive when the industry price is higher on average.

With common shocks, an increase in the number of firms that do not hedge creates a stronger disincentive for an individual firm to remain unhedged as opposed to hedging, compared to the case of idiosyncratic shocks. This is because, as discussed above, the sensitivity of the output price to the common shock increases as more firms choose not to hedge. This reduces the ability of a firm that remains unhedged to benefit from production flexibility.

To sum up, for the case of common shocks, an increase in the number of firms that remain unhedged decreases the attractiveness of remaining unhedged because the two risk exposures faced by the firm - one from a random marginal cost of production, the other from a random output price - tend to offset each other. At the same time, a firm that hedges enjoys higher profits, since it can benefit from another type of production flexibility. By being able to produce at a constant marginal cost in an environment in which the output price is random, a hedged firm can produce higher amounts compared to its unhedged competitors when the latter have high costs and prices are high, and produce lower amounts when competitors have low costs and prices are low. A greater variability in the price increases the value of this production flexibility, and increases its profits.

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19 This is easily checked by differentiating expressions (24) and (26) with respect to $m^*$. The derivative is larger in magnitude for the former expression since $E(w_i^2) - (E(w_i))^2 = \text{Var}(w_i) > 0$.

20 Nain (2004) finds that the sensitivity of industry price to exchange rate changes is lower for industries in which proportionately more firms use foreign currency derivatives. Changes in foreign exchange rates affect the cost of imports and hence the industry price; however, if more firms are involved in foreign currency hedging, the impact on industry price will be less. Nain also finds that the adverse impact of a depreciation of the US dollar on the stock returns of unhedged “import dependent” firms is higher if more firms in the industry hedge.
4 Analysis of the Equilibrium

Let us return to the case of common cash flow shocks across all firms. In this case, by virtue of Proposition (2), the proportion of firms that do not hedge in equilibrium can be obtained approximately by setting Equation (24) equal to zero.

Proposition 3 The proportion of firms that remain unhedged in equilibrium, $\frac{m^u}{n}$, lies within the following bounds:

$$\frac{1}{2} + \left( \frac{1}{n} + \frac{\delta}{nb} \right) \left[ \frac{1}{2} - \left( a - \frac{c}{\gamma(k)} \right) \theta \right] \pm \frac{1}{n} \quad (27)$$

where

$$\theta \equiv \frac{E(w)}{E(w^2)}$$

provided the bounds are within the unit interval. If the lower of the two bounds exceeds unity, all firms remain unhedged. If the higher of the two bounds is negative, all firms hedge.

Proof. Immediate from Lemma 1 and Proposition (2).

The reason why there may exist an interior solution for the number of firms that hedge/do not hedge in equilibrium is because the covariance between the output price and the random shock to marginal cost $w$ is positive. An interior solution does not exist if either the demand curve is flat (the industry is a price-taking industry), or if the cost shocks are independently distributed. In these cases, either all firms hedge or all firms remain unhedged, and we do not have any heterogeneity in firms’ hedging strategies. With a common cost shock, the sensitivity of the output price to the cost shock increases as more firms remain unhedged. As we discussed in section (3.3.1), this increases the attractiveness of hedging and decreases the attractiveness of remaining unhedged. Thus, it is possible that for a particular value of $m^u$ (the number of unhedged firms) that is between 1 and $n$, the profits from hedging and not hedging are equal (or more precisely, $m^u$ is such that a firm that is currently hedged earns
a lower profit if it deviates and chooses not to hedge, and a firm that is currently unhedged deviates and chooses to hedge), and we have an equilibrium.

Note that the equilibrium fraction of firms that hedge/do not hedge is $\frac{1}{2} \pm$ some adjustment. Let’s define

$$\theta^* \equiv \frac{1}{2(a - \frac{a}{\gamma(k)})}. \quad (28)$$

If $\theta = \theta^*$, then approximately $\frac{1}{2}$ of the firms hedge and $\frac{1}{2}$ of the firms do not, for large $n$. In this case the equilibrium is most heterogeneous, i.e., there is maximum diversity in hedging strategies. We now consider how the proportion of firms that hedge/do not hedge depends on a number of industry characteristics, such as the number of firms in the industry, the nature of competition, and the slopes of the marginal cost curve and the demand curve.

**Proposition 4** Consider an industry with a fixed number of firms. If $\theta$ is above (resp. below) the critical level $\theta^*$, then the proportion of firms that choose not to hedge

1. Is less than $\frac{1}{2} + \frac{1}{n}$ (resp. greater than $\frac{1}{2} - \frac{1}{n}$).
2. Decreases (resp. increases) as
   
   (a) the industry demand curve becomes flatter (with intercept unchanged) (i.e. as $b$ becomes smaller),
   
   (b) production flexibility decreases (i.e. as $\delta$ increases)
   
   (c) the number of firms in the industry decreases (i.e. as $n$ decreases).

**Proof.** Immediate from the expression in (27).$$^21$$

The results in Proposition (4) indicate how the proportion of firms that do not hedge (or hedge) in equilibrium changes as industry parameters change. Recall that the cost of

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$^{21}$ A more precise statement for Parts 2(a) and (b) is that both the bounds for the proportion of firms that do not hedge in equilibrium given in Proposition (27) decrease (resp. increase) as $b$ decreases or $\delta$ increases. Similarly, a more precise statement for Part 2(c) is that the lower bound in Proposition (27) decreases (resp. the upper bound in proposition (27) increases) as $n$ decreases.
remaining unhedged increases in $E(w)$, which appears in the numerator of $\theta$, while the *benefit* of remaining unhedged increases in $E(w^2)$, which appears in the denominator of $\theta$. Thus, it is intuitive that less than half the firms in the industry will choose not to hedge if $\theta$ exceeds a critical level, as implied by the first part of Proposition (4).

A useful way to understand the other parts of Proposition (4) is to realize that all parameter changes considered here (a flatter industry demand curve, a steeper marginal cost curve, and fewer firms in the industry) cause the proportion of firms following the same hedging strategy to move further away from the ratio of $\frac{1}{2}$. If $\theta > \theta^*$, the proportion of firms that do not hedge moves away from $\frac{1}{2}$ and towards 0, while if $\theta < \theta^*$, the proportion of firms that do not hedge moves away from $\frac{1}{2}$ and towards 1, as the industry demand curve becomes flatter, the marginal cost curve steeper, or there are fewer firms in the industry. Thus, they lead to less heterogeneity in hedging strategies. Whether the equilibrium entails the majority of firms hedging or not hedging, depends on the market size, measured by the ratio of the demand intercept $a$ to the intercept of the marginal cost curve $c$.\footnote{For given $n$, a larger market is synonymous with a larger market share.} If the market is relatively large, $\theta^*$ will be small, so with $\theta > \theta^*$, a majority of firms will hedge. If the market is relatively small, a majority of firms will not hedge. However, under the parameter changes indicated in Proposition (4), more firms will join the majority, and the equilibrium will become less heterogenous.

To understand why the parameter changes have this implication, notice that a change in the parameters has two effects on a firm’s incentive to hedge. Let us first consider the effect of a flatter (more elastic) industry demand curve, holding the intercept of the demand
curve $a$ unchanged. A flatter industry demand curve implies that the output price is less volatile. This will increase the incentive for an individual firm to remain unhedged in order to maximize the value of its production flexibility. The unhedged firm benefits because it produces more when its marginal cost is low and less when its marginal cost is high. The second effect of a flatter demand curve (with intercept unchanged) is that the average industry price will be higher. If the industry price is higher on average, it is more costly to have a higher expected marginal cost of production. Thus, since $E(w) > 0$, remaining unhedged is more costly.

Thus, there are two opposing effects of a decrease in the slope of the demand curve: one increases the incentive to remain unhedged, while the other decreases that incentive. Which of these two effects dominates depends on the market size, i.e., the demand intercept $a$ in relation to the intercept of the marginal cost curve $c$. If the market size is sufficiently small ($\theta < \theta^*$), then firms operate in the convex part of their profit functions, and remaining unhedged is more attractive. Thus, irrespective of the other parameters, more than half of the firms choose not to hedge. What our results show is that, in this situation, the lower sensitivity of the industry price to unhedged firms’ investment levels will have a more dominant effect on the incentive to remain unhedged than the offsetting effect of a higher average industry price. The latter effect is less important because the average industry price is still relatively low if the market size is small, and therefore the loss associated with the higher

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23 To see this, consider a price taking firm with cost function $C(q) = \frac{\delta}{2}q^2 + \hat{c}q$, where $\hat{c}$ is a random shock that affects marginal cost. Assume output is chosen after the cost shock is realized. Then, the profit conditional on a particular value of the realized $\hat{c}$ is $\Pi(\hat{c}) = (P - \hat{c})q$, and the expected profit is $E\Pi(\hat{c}) = \frac{1}{E\hat{c}}[P^2 + E(c^2) - 2PE(\hat{c})]$. Notice that the expected profit is increasing in $E(c^2)$ (the benefit of flexibility) and $\frac{dE\Pi(\hat{c})}{dE(\hat{c})} = -\frac{\delta q}{E\hat{c}}$. i.e. the loss in profit from a higher expected intercept of the marginal cost curve is proportional to the price.
expected marginal cost of production from remaining unhedged is not very high. Exactly the opposite holds when the market size is relatively large ($\theta > \theta^*$), so that a flatter demand curve encourages more hedging even if more than half of the firms choose to hedge already. It is easy to see that the argument with respect to the other parameter changes considered in Proposition 2 is essentially the same as that for the slope of the industry demand curve.

In contrast to the results of Proposition (4), the following results do not depend on the critical level $\theta^*$.

**Proposition 5** The bounds for the proportion of firms that choose not to hedge in expression (27)

1. decrease as $\theta$ increases,

2. decrease as $a$, the intercept of the demand curve, increases.

**Proof.** Immediate from the expression in (27). ■

The intuition for these results is as follows. An increase in $\theta$, or an increase in the average market price (due to a higher demand intercept $a$), makes it more costly for a firm not to hedge, as discussed above. Hence, the proportion of firms that choose not to hedge decreases.\(^{24}\)

There is a limited amount of empirical evidence on how industry characteristics affect the hedging behavior of firms. As shown in Figure 1, Allayannis and Weston (1999) find that firms that operate in industries with lower price-cost margins were more likely to use foreign

\(^{24}\) Nain (2004) notes that hedgers tend to be larger firms. With fixed $n$, an increase in $a$ implies a higher market share for each firm. Thus, the proportion of hedgers will be higher among firms with higher market share.
currency derivatives between 1994 and 1995. Notice that ceteris paribus, a higher number of firms in the industry would imply a lower price-cost margin. An interesting feature of the Allayannis and Weston (1999) sample is that in most of the industries that they consider, the percentage of firms using derivatives is less than $\frac{1}{2}$. Equation (27) implies that Allayannis and Weston (1999) mostly consider industries in which $\theta < \theta^*$, so that the proportion of firms that hedge is less than $\frac{1}{2}$. The same equation also implies that when $\theta < \theta^*$, a higher price-cost margin (a smaller number of firms) will be associated with a smaller proportion of firms that hedge, i.e., there is a negative relationship between the price-cost margin and the proportion of firms that hedge, consistent with Allayannis and Weston (1999).

Our results are also consistent with those reported by Geczy, Minton and Schrand (1997). Data presented by these authors (see Figure 1) implies that there is an inverted u-shaped relationship between the number of firms in each industry and the proportion of firms that use derivatives. This, too, is consistent with our model (equation (27)). When the proportion of firms that hedge is less than $\frac{1}{2}$ (i.e., when $\theta < \theta^*$), the relationship between $n$ and the proportion of firms that hedge is positive; however, the relation is negative if the proportion of firms that hedge exceeds $\frac{1}{2}$ ($\theta > \theta^*$). Thus, it is clear from Propositions (4) and (5) that tests of this nature could be extended in a number of ways to gain additional insight about the role of industry characteristics on the incentives for firms to hedge.25

Our analysis has so far focused exclusively on a situation in which all firms in the industry are financially constrained. However, in any given industry, firms will differ in the extent to

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25 For example, with a flat industry demand curve, hedging strategy will be homogeneous, and will switch between all (most) financially constrained firms hedging when the price is high, to few (none) hedging when the price is low. Export industries experiencing exchange rate fluctuations that affect the product price would appear to be good candidates for tests of this nature.
which they are constrained. A larger number of firms that are financially *unconstrained* is equivalent, in our model, to the financially constrained firms facing a lower residual demand curve, i.e., a lower intercept of the demand curve \( a \). One implication of this result would be that the larger the proportion of sales in an industry that is attributable to, say, the top four firms, the lower would be the proportion of the remaining firms that hedge, ceteris paribus.

5 Continuous Hedging Strategies

Up to this point we have assumed that firms follow discrete hedging strategies, i.e., firms either completely hedge their cash flows or they do not hedge at all. Now we assume that firms can choose from a continuum of hedging strategies, which we model as follows. Let \( h \in [0, 1] \) represent a firm’s hedge ratio. Then the firm’s cash flow at time \( T=1 \) is given by

\[
y_h = (1 - h)y + h\bar{y}.
\]

Unfortunately, a closed-form solution for the general model with \( n \) firms and a continuum of states does not exist. However, we can solve the problem for a particular example and show that asymmetric equilibria exist even when the strategy space is an interval.

We proceed as follows: We first consider an example with only two firms and two states of nature. For this case we can explicitly depict the reaction functions of the two firms. We show that for certain parameter values the only equilibria are corner solutions. The corner solutions either involve both firms hedging \((h_i = 1, i = 1, 2)\), both firms remaining unhedged \((h_i = 0, i = 1, 2)\), or one firm hedging and the other remaining unhedged \((h_i = 1, h_j = 0, i \neq j, i = 1, 2)\). Thus, as in the equilibrium corresponding to Proposition (3), heterogeneity of hedging strategies is possible even when hedging strategies are continuous and the firms are
ex-ante symmetric. We show that which of these equilibria prevails depends on the market size, i.e., the magnitude of $a$. In particular, consistent with the analysis in the previous section (Proposition (5)), a higher $a$ can cause a shift from both firms remaining unhedged or only one firm hedging to both firms hedging in equilibrium.

Second, we extend the example to a larger number of firms ($n = 20$). The parameters are chosen such that, when the $h$ for every firm is restricted to the values of 0 and 1, the equilibrium of the previous section would imply that exactly 10 firms will hedge, and the other 10 will remain unhedged. We show that this particular equilibrium remains when $h$ is allowed to take any value in the unit interval. Thus, all our earlier results go through when a continuum of hedging strategies is allowed.

5.1 Example 1

Let $n = 2$, $a = 4.3335$, $c = 1$, $\delta = 0.5$, and $b = 0.5$. The random variable $k$ can take on one of two values: $k = 0.25$ or $k = 0.5$, with equal probabilities. Furthermore, let $\gamma(k) = k$.

First, we look for symmetric equilibria. In Appendix C, we show that for any symmetric equilibrium $(h, h)$ in which firm 1’s $h$ is a unique best response to the other firm’s choice of $h$, the following first-order condition must be satisfied:

$$
0 = \left(4.3335 - \frac{1}{(1-h)(0.25) + (0.375)h}\right) \left(\frac{1}{((1-h)(0.25) + (0.375)h)^2}\right) - \left(4.3335 - \frac{1}{(1-h)(0.5) + (0.375)h}\right) \left(\frac{1}{((1-h)(0.5) + (0.375)h)^2}\right)
$$

This equation has two solutions in $h \in [0, 1]$, which are: $h = 1.0$ and $h = 0.2722$. However, by fixing the $h$ of one of the firms at each value and plotting the expected profit of the other firm as a function of its own $h$, we are able to check that both are points of
local minima, so that the second order conditions for optimal response are not satisfied. This implies that a symmetric equilibrium cannot exist. However, because the reaction functions are symmetric, it follows that they must be discontinuous (otherwise, a symmetric point of intersection would exist).

In Figure 2, we plot the reaction functions of the two firms. As the above reasoning suggests, there is a discontinuity in each reaction function at \( h = 0.434 \). Specifically, if Firm 1 chooses \( h = 0.434 \), then Firm 2 is indifferent between \( h = 0 \) and \( h = 1 \). If Firm 2 chooses \( h = 0.434 \), then Firm 1 is indifferent between \( h = 0 \) and \( h = 1 \). The reaction functions further reveal that there are two corner equilibria, in which one firm hedges completely and the other firm remains completely unhedged.

Figure 3 shows how the equilibrium changes if the demand intercept \( a \) increases to 6.0. The unique symmetric equilibrium is at the corner, with both firms choosing \( h = 1 \). This is consistent with the comparative static results of Proposition (5), which states that the number of firms that hedge increases as \( a \) increases.27

5.2 Example 2

Let \( n = 20, a = 4.3335, c = 1, \delta = 0.5, \) and \( b = 0.5 \). The random variable \( k \) can take on one of two values: \( k = 0.25 \) or \( k = 0.5 \), with equal probabilities. Furthermore, let \( \gamma(k) = k \).

The only difference between this example and the previous one is that the number of firms in the industry is 20. In Appendix C we verify that for a symmetric equilibrium to

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26 The equation corresponding to the first-order condition analogous to equation (30) still has a solution at \( h = 1 \), but now this corresponds to a local maximum.

27 While we do not show a plot of this case, for sufficiently low \( a \), the reaction functions coincide with the axes, and the unique equilibrium is \((0, 0)\), i.e., neither firm hedges.
exist the first-order condition (30) must hold again. As before, the solutions to this equation correspond to local minima, and the reaction functions are discontinuous at a symmetric interior value. Thus, no symmetric equilibria exist.

Given the above parameter values, it can be verified that when the \( h \) is restricted to be either 0 or 1, half the firms hedge and half the firms remain unhedged in equilibrium. We plot the expected profit of an individual firm as a function of its own hedging strategy \( h \), assuming that 10 firms hedge completely (\( h = 1 \)) and the remaining 9 firms hedge not at all (\( h = 0 \)). Figure 4(a) shows that there is no maximum in the interior of the interval \([0, 1]\). In fact, not hedging (\( h = 0 \)) is the best response for this firm. Similarly, plot the expected profit of an individual firm as a function of its own hedging strategy \( h \), assuming that 10 firms do not hedge (\( h = 0 \)) and the remaining 9 firms hedge completely (\( h = 1 \)). Figure 4(b) shows that in this case too there is no interior optimum. The firm’s best response is to completely hedge (\( h = 1 \)). Thus, the corner equilibrium in which 10 firms hedge completely (\( h = 1 \)) and 10 firms remain completely unhedged (\( h = 0 \)) survives even if the hedging strategy space is the unit interval. By varying the parameter values, we checked that the equilibria corresponding to the discrete case survive for these parameter values as well. Thus, all comparative static results from the previous section remain valid if firms can partially hedge.

5.3 **Allowing \( h \) outside the \([0, 1]\) interval**

So far we have assumed that \( h \) can take on any value in the interval \([0, 1]\). We now relax this assumption to some extent, although it seems reasonable to restrict \( h \) for two reasons. First, company policies may prevent managers from taking positions that could be interpreted as speculative. Second, managers may not like extreme variability in their cash flows. As we will
see below, if $h$ is negative or if $h$ significantly exceeds unity, then a firm may have problems meeting its fixed payment obligations if its cash flow shock turns out to be low. Moreover, if $h$ is larger than 1, then the actual and hedged cash flows are negatively correlated.\footnote{Note that $E(y - \bar{y})(y_n - E(y_n)) = E(y - \bar{y})(y_n - \bar{y}) = E(y - \bar{y})((1 - h)y + h\bar{y} - \bar{y}) = (1 - h)E(y - \bar{y})^2.$} The ability of firms to attain such negative correlation may be limited.

For illustration, let us return to Example 1 and assume that each firm has some fixed payment obligation $F$ equal to 0.25. If the cost of defaulting on this fixed payment obligation is sufficiently high, then the firm will ensure that its cash flow will never fall below 0.25. This implies that $h \in [0, 2]$\footnote{From $0.25 \leq (1 - h)(0.25) + h(0.375)$ we get $h = 0$ as the lower bound on $h$. Solving $0.25 = (1 - h)(0.5) + h(0.375)$ gives $h = 2.$} If we further assume that each firm has an investment of 0.25 already in place, then each firm’s capital stock is still

$$y - F + 0.25 = y - 0.25 + 0.25 = y = k(1 - h) + \bar{kh}.$$  

Figure 5 illustrates the reaction functions assuming $h \in [0, 2]$. Now $(2, 0)$ and $(0, 2)$ are asymmetric equilibria. There is a discontinuity at $h = 1$, at which the firms are indifferent between $h = 2$ and $h = 0$. Figure 6 illustrates the reaction functions if the demand intercept is increased to $a = 6$. In this case $(1, 1)$ is an equilibrium.\footnote{Two further asymmetric equilibria are $(0, 2)$ and $(2, 0).$} Thus, an increase in $a$ allows the possibility of an equilibrium in which both firms hedge.

What these examples show is that, much like our earlier analysis, an increase in $a$ can lead to more firms hedging in equilibrium. Furthermore, the multiplicity of equilibria illustrates why it might be difficult to find hedging practices to be systematically related to firm or industry characteristics.
6 Conclusion

The existing literature suggests that financially constrained firms have an incentive to hedge so as to increase the correlation between their cash flows and investment expenditures. Our analysis contributes to this literature by showing that under fairly standard assumptions the profit function of a financially constrained firm may be convex in the level of investment. This implies that financially constrained firms may have an incentive to speculate rather than to hedge. A further contribution of our model is that it shows that a firm’s incentive to hedge depends partially on the hedging choices of other firms in the same industry. Specifically, the incentives of an individual firm to hedge decreases as more firms hedge, and increases as more firms choose not to hedge. In the spirit of Maksimovic and Zechner (1991) and Shleifer and Vishny’s (1992) we show that the proportion of firms that hedge is determined within the equilibrium, and that in equilibrium, individual firms are indifferent between hedging and not hedging.

Our analysis helps us understand why there is such diversity in risk management strategies even within the same industry. Tufano (1996) suggests that this diversity is due to differences in managers’ compensation packages, which provide different incentives to hedge. This explanation, however, begs the question of why shareholders would give their managers different incentives to hedge if hedging is deemed beneficial. Our analysis suggests that the diversity in hedging strategies can be a feature of the equilibrium, even if all firms are identical ex ante. A larger fraction of firms will hedge, however, in industries that are subject to tighter financial constraints and industries with larger markets.

While there is strong evidence of heterogeneity in hedging choices in specific industries,
such as gold mining and oil and gas, there has not been a more general study of heterogeneity in hedging choices across all industry sectors. Our analysis predicts that in industries with more competition, more inelastic demand, and less convexity in production costs we should expect to observe more heterogeneity in hedging choices.
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Appendix A

Long run average cost curves (as functions of output $q$) implied by the class $\gamma(k) = k^\phi$ are U-shaped.

To see this, notice that the long run cost function is

$$C(q) = \frac{1}{2} \delta q^2 + \frac{q}{k^\phi} + k. \quad (31)$$

where $k$ is chosen to minimize the above expression for each $q$. The level of $k$ that minimizes this expression for given $q$ satisfies the first-order condition

$$1 = \phi k^{-(1+\phi)} q. \quad (32)$$

Thus, the long run average cost is

$$LAC(q) = \frac{1}{2} \delta q + k^{-\phi} + k q^{-1}$$
$$= \frac{1}{2} \delta q + (\phi^{-\phi} + \phi^{-\phi}) q^{-\phi} \quad (33)$$

where the second line is obtained by substituting for $k$ from equation (32). It is easily checked that the expression is first decreasing and then increasing in $q$. □
Appendix B

Proof of Lemma 1:

We have

\[ q^u = \frac{P(w, m^u) - w - \alpha}{\delta + b}, \]

\[ q^h = \frac{P(w, m^u) - \alpha}{\delta + b} \]

and

\[ Q = \left[ q^u m^u + q^h m^h \right]. \] (34)

Substituting for the expressions for \( q^u \) and \( q^h \), we have

\[ Q = \frac{(P(w, m^u) - \alpha - w)m^u + (P(w, m^u) - \alpha)m^h}{\delta + b}. \] (35)

Substituting the expression for output for the unhedged firm back into the expression for the profit in equation (21), we get

\[ \Pi^u(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} (P(w, m^u) - w - \alpha)^2. \] (36)

Similarly, for the hedged firm, we get

\[ \Pi^h(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} (P(w, m^u) - \alpha)^2. \] (37)

Taking expectations, and subtracting, we get

\[ E\Pi^u(w, m^u) - E\Pi^h(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} [E(w^2) - 2E \{(P(w, m^u) - \alpha)w\}]. \] (38)

Substituting for \( Q \) from equation (15) and solving for \( P \), we get

\[ P(w, m^u) - \alpha = \frac{(a - \alpha)(\delta + b) + bm^u w}{nb + \delta + b}. \] (39)

Thus,

\[ (P(w, m^u) - \alpha)w = \frac{(a - \alpha)(\delta + b)}{nb + \delta + b}w + \frac{bm^u}{nb + \delta + b}w^2. \] (40)

Hence

\[ E \{(P(w, m^u) - \alpha)w\} = \frac{(a - \alpha)(\delta + b)}{nb + \delta + b}E(w) + \frac{bm^u}{nb + \delta + b}E(w^2). \] (41)

Substituting from equation (41) into equation (38), we get

\[ E\Pi^u(w, m^u) - E\Pi^h(w, m^u) = \frac{(\delta + 2b)}{2(\delta + b)^2} \left[ E(w^2) - \frac{2(a - \alpha)(\delta + b)}{nb + \delta + b}E(w) - \frac{2bm^u}{nb + \delta + b}E(w^2) \right]. \]

\[ \square \]
Derivation of Equation (26):

In the case of idiosyncratic shocks, let \( w_i \) denote the shock to firm \( i \), and \( \mathbf{w} \) denote the vector \((w_1, w_2, \ldots, w_n)\). The output of a firm that does not hedge can be derived as before to be \( q_i^n = \frac{P(w,m^u) - w_i - \alpha}{\delta + b} \), while that of a firm that hedges is \( q_i^h = \frac{P(w,m^u) - \alpha}{\delta + b} \). Equilibrium in the product market implies

\[
\frac{a - P(w,m^u)}{b} = \sum_{m^u} \left( \frac{P(w,m^u) - w_i - \alpha}{\delta + b} \right) + \sum_{m^h} \left( \frac{P(w,m^u) - \alpha}{\delta + b} \right).
\]

Simple manipulations give

\[
P(w,m^u) - \alpha = \frac{(a - \alpha)(\delta + b) + b\sum w_i}{nb + \delta + b}.
\]

Hence,

\[
E\{(P(w,m^u) - \alpha)w_i\} = \frac{(a - \alpha)(\delta + b)}{nb + \delta + b} E(w_i) + \frac{b(m^u - 1)}{nb + \delta + b}(E(w_i))^2 + \frac{b}{nb + \delta + b} E(w_i^2)
\]

since \( \text{Cov}(w_i, w_j) = 0 \Rightarrow E(w_i, w_j) = (E(w_i))^2 \) if \( i \neq j \). It is easily checked that

\[
\begin{align*}
\text{EII}^u(w,m^u) - \text{EII}^h(w,m^u) & = \frac{\delta + 2b}{2(\delta + b)^2} \left[ E(w_i^2) - 2E\{(P(w,m^u) - \alpha)w_i\} \right] \\
& = \frac{\delta + 2b}{2(\delta + b)^2} \left[ E(w_i^2) - 2\frac{(a - \alpha)(\delta + b)}{nb + \delta + b} E(w_i) - \frac{2b(m^u - 1)}{nb + \delta + b}(E(w_i))^2 - \frac{2b}{nb + \delta + b} E(w_i^2) \right]
\end{align*}
\]

\( \square \)

Proof of Proposition (2):

The proof uses the following Lemma:

Lemma A1: (i) \( \text{EII}^h(w,m^u) \) is increasing in \( m^u \). (ii) \( \text{EII}^u(w,m^u) \) is decreasing in \( m^u \) for \( m^u < \frac{n+1}{2} + \bar{m}^u \), where \( \bar{m}^u \) is the real number value of \( m^u \) for which the right hand side of (24) is zero.

Proof: (i) We have from (37)

\[
\text{EII}^h(w,m^u) = \frac{\delta + 2b}{2(\delta + b)^2} E(P(w,m^u) - \alpha)^2
\]

\[
= \frac{\delta + 2b}{2(\delta + b)^2} \left[ \text{constant} + \frac{b^2(m^u)^2}{(nb + \delta + b)^2} E(w_i^2) + \frac{2(a - \alpha)(\delta + b)m^u}{(nb + \delta + b)^2} E(w) \right].
\]

where we have used equation (39). This is increasing in \( m^u \).

(ii) We have, from (36):

39
 unhedged, since $E$ to deviate and become hedged, since inequality follows from Lemma A1 and the second from the definition of $m^u$. Similarly, none of the remaining $n$ is unhedged. Again, since $E$ unhedged, since there is no incentive for any of the remaining $n - m^u$ firms to deviate and become hedged, since its payoff $E$ unhedged.

Using (39) to substitute for $E(P(w, m^u) - \alpha)^2$ and using (41), we get:

$$E \Pi^u(w, m^u) = \frac{\delta + 2b}{2(\delta + b)^2} E(P(w, m^u) - w - \alpha)^2.$$}

Treating $m^u$ as a real number and differentiating with respect to $m^u$, we get:

$$\frac{dE \Pi^u(w, m^u)}{dm^u} = -\frac{b(\delta + 2b)}{2((n + 1)b + \delta)(\delta + b)^2} \left[2E(w^2) - \frac{2(a - \alpha)(\delta + b)}{nb + \delta + b} E(w) - \frac{2bm^u}{nb + \delta + b} E(w^2)\right]$$

$$-\frac{b(\delta + 2b)}{2((n + 1)b + \delta)(\delta + b)^2} \left[\left\{E(w^2) - \frac{2(a - \alpha)(\delta + b)}{nb + \delta + b} E(w) - \frac{2bm^u}{nb + \delta + b} E(w^2)\right\}\right].$$

Notice that the expression within curly brackets is zero, from the definition of $\overline{m}^u$. Thus, the bracketed expression is positive when $1 > \frac{2b(m^u - \overline{m}^u)}{nb + \delta + b}$. Hence, the result follows. □

**Proof of Proposition:**

First Part. If an integer $m^u$ exists such that the right hand side of (24) is zero, then none of the remaining $n - m^u$ firms that are hedged have any incentive to deviate and become unhedged, since $E \Pi^u(w, m^u + 1) < E \Pi^u(w, m^u) = E \Pi^b(w, m^u)$ at this value of $m^u$, from Lemma A1. Similarly, none of the $m^u$ firms that are currently unhedged have any incentive to deviate and become hedged, since $E \Pi^b(w, m^u - 1) < E \Pi^b(w, m^u) = E \Pi^u(w, m^u)$ from Lemma A1. Thus, the necessary and sufficient conditions for a SPNE are satisfied for $m^u$.

Second Part: (a) If $E \Pi^u(w, m^u_2) < E \Pi^b(w, m^u_1)$, then given that $m^u_1$ firms are unhedged, there is no incentive for any of the remaining $n - m^u_1$ firms that are hedged to become unhedged. Again, since $E \Pi^b(w, m^u_1 - 1) < E \Pi^b(w, m^u_1) < E \Pi^u(w, m^u_1)$ (where the first inequality follows from Lemma A1 and the second from the definition of $m^u_1$), none of the currently unhedged firms has any incentive to deviate and become hedged. Thus, the necessary and sufficient conditions for a SPNE are satisfied for $m^u_1$.

(b) If $E \Pi^u(w, m^u_2) > E \Pi^b(w, m^u_1)$, then none of the $m^u_2$ unhedged firms has any incentive to deviate and become hedged, since its payoff $E \Pi^b(w, m^u_2) < E \Pi^u(w, m^u_2)$. Similarly, none of the remaining $n - m^u_2$ firms that are hedged has any incentive to deviate and become unhedged, since $E \Pi^u(w, m^u_2 + 1) < E \Pi^u(w, m^u_2) < E \Pi^b(w, m^u_2)$, where the first inequality follows from Lemma A1 and the second from the definition of $m^u_2$. Thus, the necessary and sufficient conditions for a SPNE are satisfied at $m^u_2$. □
Appendix C

Derivation of First-Order Necessary Condition for Symmetric Hedging Equilibrium when $h$ is continuous

We confine ourselves to two states (or realizations of $y = k$), which we denote by $A$ and $B$. The probability of these states is denoted by $\theta^A$ and $1 - \theta^A$, respectively. Let the investment corresponding to these states be denoted by $k^A$ and $k^B$, respectively. Also, let $\gamma(k) = \frac{1}{2}$, $b = 0.5$, $c = 1$ and $\delta = 0.5$.

Firm $i$ chooses $h_i$ to maximize

$$\theta^A \Pi^A_i(q^A(h)) + (1 - \theta^A) \Pi^B_i(q^B(h))$$

where

$$\Pi^A_i(q^A(h)) = \left( a - b \left( q^A_i(h_1, \cdots, h_n) + \sum_{j \neq i} q^A_j(h_1, \cdots, h_n) \right) \right) q^A_i(h_1, \cdots, h_n)$$

$$- \frac{1}{2} \left( q^A_i(h_1, \cdots, h_n) \right)^2 \frac{1}{(1 - h_i)k^A + h_i k} q^A_i(h_1, \cdots, h_n)$$

and $q^A(h) \equiv (q^A(h_1, \cdots, h_n), \cdots, q^A_n(h_1, \cdots, h_n))$. Variables superscribed with $B$ are similarly defined.

The first-order necessary condition for optimal choice of $h_i$ is:

$$\theta^A \left( \sum \frac{\partial \Pi^A_i}{\partial q^A_j} \frac{\partial q^A_j}{\partial h_i} \right) + (1 - \theta^A) \left( \sum \frac{\partial \Pi^B_i}{\partial q^B_j} \frac{\partial q^B_j}{\partial h_i} \right) = 0. \quad (44)$$

The first order conditions for output choice for firm $i$ in state $A$ are given by

$$a - (1.5)q^A_i - (0.5) \sum_{j \neq i} q^A_j - \frac{1}{(1 - h_i)k^A + h_i k} = 0. \quad (45)$$

Similarly, for $k \neq i$, the first-order condition is analogously obtained as:

$$a - (1.5)q^A_k - (0.5) \sum_{j \neq i,k} q^A_j - (0.5)q^A_i - \frac{1}{(1 - h_k)k^A + h_k k} = 0 \quad , k \neq i, k = 1, \cdots, n. \quad (46)$$

Differentiating these first-order-conditions with respect to $h_i$, one gets:

$$-(1.5) \frac{dq^A_i}{dh_i} - (0.5) \sum_{j \neq i} \frac{dq^A_j}{dh_i} + \frac{\bar{k} - k^A}{(1 - h_i)k^A + h_i k} = 0 \quad (46)$$

$$-(1.5) \frac{dq^A_k}{dh_i} - (0.5) \sum_{j \neq i,k} \frac{dq^A_j}{dh_i} - (0.5) \frac{dq^A_i}{dh_i} = 0 \quad , k \neq i, k = 1, \cdots, n. \quad (47)$$
In a symmetric equilibrium, let us denote \( \frac{dq^A}{dh_i} = \alpha_1^A \) and \( \frac{dq^A}{dh_i} = \alpha_2^A \) for \( j \neq i \). The above equations then reduce to

\[
-(1.5)\alpha_1^A - \frac{n-1}{2}\alpha_2^A + \frac{\bar{k} - k^A}{((1-h)k^A + hk)^2} = 0 \quad (48)
\]

\[
-(0.5)\alpha_1^A - \frac{n+1}{2}\alpha_2^A = 0 \quad (49)
\]

Solving, we get:

\[
\alpha_2^A = -\frac{1}{n+2} \frac{\bar{k} - k^A}{((1-h)k^A + hk)^2}. \quad (50)
\]

Exactly similarly, we can define \( \frac{dq^B}{dh_i} = \alpha_2^B \) and we get

\[
\alpha_2^B = -\frac{1}{n+2} \frac{\bar{k} - k^B}{((1-h)k^B + hk)^2}. \quad (51)
\]

In symmetric equilibrium, \( q_j^A = q^A \) for all \( j = 1, \cdots, n \) and \( q_j^B = q^B \) for all \( j = 1, \cdots, n \). Substituting this in the first-order conditions \((?\?)\) (and an analogous set of conditions for state \(B\)), we get

\[
q^A = \frac{2}{n+2} \left( a - \frac{1}{((1-h)k^A + hk)} \right) \quad (52)
\]

\[
q^B = \frac{2}{n+2} \left( a - \frac{1}{((1-h)k^B + hk)} \right) \quad (53)
\]

Now notice that, using the envelope theorem, the first-order condition \((44)\) reduces to

\[
-\theta^A b q_i^A \alpha_2^A - (1 - \theta^A) b q_i^B \alpha_2^B = 0. \quad (54)
\]

Substituting, we get

\[
0 = \theta^A \left( a - \frac{1}{((1-h)k^A + hk)} \right) \left( \frac{\bar{k} - k^A}{((1-h)k^A + hk)^2} \right) + (1 - \theta^A) \left( a - \frac{1}{((1-h)k^B + hk)} \right) \left( \frac{\bar{k} - k^B}{((1-h)k^B + hk)^2} \right) \quad (55)
\]

Notice that the first-order condition for symmetric equilibrium is independent of the number of firms, \(n.\square\)
**Industry Mark-Ups**

(Value of sales + change in inventories - payroll - cost of materials) / (Value of sales + change in inventories)

*Source: Allayannis and Weston (1999)*

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**Number of Firms in Industry (Sample)**

*Source: Geczy, Minton and Schrand (1997)*

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Figure 1: Empirical relations between the fraction of firms that hedge and industry characteristics
Figure 2: Reaction Fuctions for $n = 2$, $a = 4.3335$, $c = 1$, $b = 0.5$, $\beta = 1$, $\delta = 0.5$ and $\gamma(k) = k$. 
Figure 3: Reaction Functions for $n = 2$, $a = 6.0$, $c = 1$, $b = 0.5$, $\beta = 1$, $\delta = 0.5$ and $\gamma(k) = k$. 
Figure 4(a): $n = 20$, $a = 4.3335$, $c = 1$, $b = 0.5$, $\beta = 1$, $\delta = 0.5$ and $\gamma(k) = k$. Figure shows a firm’s Expected Profit as a Function of $h$. There are 10 firms with $h = 1$, 9 firms with $h = 0$. 
Figure 4(b): \( n = 20, a = 4.3335, c = 1, b = 0.5, \beta = 1, \delta = 0.5 \) and \( \gamma(k) = k \). Figure shows a firm’s Expected Profit as a Function of \( h \). There are 9 firms with \( h = 1 \), 10 firms with \( h = 0 \).
Figure 5: Reaction Fuctions for $n = 2$, $a = 4.3335$, $c = 1$, $b = 0.5$, $\beta = 1$, $\delta = 0.5$ and $\gamma(k) = k$. $h$ is allowed to be in range $[0, 2]$. 
Figure 6: Reaction Functions for $n = 2$, $a = 6.0$, $c = 1$, $b = 0.5$, $\beta = 1$, $\delta = 0.5$ and $\gamma(k) = k$. $h$ is allowed to be in range $[0, 2]$. 