Time-Varying Credit Risk and Liquidity Premia in Bond and CDS Markets

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ABSTRACT

We develop a reduced-form credit risk model that incorporates illiquidity in the bond and the credit default swap (CDS) market. As CDS are derivative contracts, the effect of illiquidity has to be modeled differently than for bonds. For bonds, illiquidity results in yield premia while for CDS, the bid and ask quotes contain a liquidity signal. The model allows us to decompose bond yield spreads and CDS premia into their credit risk and liquidity components and to analyze the relation between credit risk and liquidity premia as well as the liquidity spill-over between the bond and the CDS market.
Introduction

Credit derivatives markets provide a standardized alternative to bond markets in taking on and selling off credit risk exposures. This development offers a new approach to one of the most widely explored problems in fixed income analysis - the separation of the corporate bond spread into its credit risk and liquidity component. The corporate bond yield spread is usually defined as the difference between the bond’s yield-to-maturity and a given default-free interest rate such as the swap rate or the yield on government bonds of the same maturity. Unarguably, credit risk is one of the spread’s most important determinants, but Elton, Gruber, Agrawal, and Mann (2001) and Collin-Dufresne, Goldstein, and Martin (2001) provide clear empirical evidence that liquidity also has a significant impact. This evidence shows that the separation of the total bond spread into its credit risk and liquidity component constitutes a central question if an issuer’s credit risk has to be quantified. Obviously, the identification of the pure credit risk component is difficult since only the sum of the two risk premia can be observed in the market.

We contribute to the existing literature on the components of bond spreads and credit default swap (CDS) premia theoretically and empirically by exploring the idea that the bid and ask quotes for CDS premia contain information on the liquidity of the CDS market. In the theoretical part of our analysis, we extend the reduced-form credit-risk model by Longstaff, Mithal, and Neis (2005) to incorporate illiquidity both in the bond and the CDS market. In the bond market, illiquidity results in price discounts and yield surcharges. This assumption is also made by Longstaff, Mithal, and Neis (2005). Our extension consists of the modeling of a twofold liquidity effect on CDS premia. First, the bond-specific liquidity has a direct effect on CDS premia since the potentially illiquid bond is delivered under the CDS contract if default occurs. Therefore, the CDS premium in our model accounts for bond liquidity as a source of bond price variation. In addition to this straightforward liquidity spill-over, we include a CDS-specific liquidity which has a more intricate effect. We circumvent the question of systematic liquidity premia in CDS mid premia by modeling the ask and bid premia instead. From these, we infer a theoretical time-varying pure credit risk CDS premium which is unaffected by the CDS-specific liquidity. Our measures of pure liquidity and of the correlation between credit risk and liquidity arise as the difference between this liquidity-free CDS premium and the mid premium. As the credit risk and liquidity premia only depend on underlying specific state variables, our model allows us to analyze in a consistent way the empirical relationship between time-varying bond- and CDS-specific liquidity premia. To the best of our knowledge, we are the first to explore this dynamic
relationship in a model of bond and CDS liquidity. Our results on the behavior of the liquidity premia can be consistently interpreted by demand relations for credit risk between the bond and the CDS market.

In the empirical part of our analysis, we separate the bond spreads and CDS premia into the pure credit risk, the pure liquidity, and the correlation-induced components for firms from a broad range of sectors and rating classes that were observed between June 1, 2001 and June 30, 2007. We then analyze the relation between the time-varying credit risk, liquidity, and correlation premia for the two markets.

Our most important findings are threefold. First, we find that adding a CDS-specific liquidity component to the model has the important consequence of consistently positive credit risk and liquidity premia in corporate bond markets. This result contrasts with that in Longstaff, Mithal, and Neis (2005) who also obtain negative liquidity premia in corporate bond yields. In particular, we show that neglecting CDS-specific liquidity can result in negative bond liquidity premia. The average bond liquidity premia for corporate firms are of a similar magnitude as the liquidity risk premia which De Jong and Driessen (2005) identify for expected excess bond returns. The CDS liquidity premium is mostly positive. This finding points to a demand pressure in the CDS market which supports the cross-sectional results of Chen, Cheng, and Wu (2005), Bongaerts, De Jong, and Driessen (2007), and Meng and ap Gwilym (2006). Overall, we attribute 60% of the total bond spread to credit risk, 35% to liquidity, and 5% to the correlation between credit risk and liquidity. These results stand in sharp contrast to those of Elton, Gruber, Agrawal, and Mann (2001), and Huang and Huang (2003) who report that the non-default component accounts for the largest percentage of the bond spread.

In the CDS market, the credit risk component constitutes 95% of the observed mid premium, the pure liquidity component 4%, and the correlation component 1%. The average liquidity premia increase for firms with higher credit risk in both markets. Our results indicate a remarkably higher CDS market liquidity than those of Tang and Yan (2007) who obtain an average liquidity premium of 13.2 basis points in their regression analysis which is similar to the Treasury bond liquidity premium of Longstaff (2004), and the average non-default component of Longstaff, Mithal, and Neis (2005).

Second, our model allows us to analyze the relation between credit risk and liquidity premia in the bond and the CDS market. In an analysis of the credit risk, liquidity, and correlation premia, we find that the bond market’s liquidity dries up as the firm’s credit risk increases. This empirical result in our reduced-form model setting supports the theoretical prediction of the structural-form model by Ericsson and Renault (2006).
They assume that liquidity shocks to the bond holder are correlated with default risk. In the CDS market, the dynamics of the liquidity premia depend on the rating class. The investment grade sector becomes more dominated by protection sellers during times of high default risk. For the subinvestment grade sector, increasing credit risk coincides with a lower demand pressure for credit protection, thus decreasing CDS liquidity premia in the subinvestment grade sector. This analysis complements the cross-sectional evidence by Dunbar (2007) who calibrates a reduced-form model with credit and liquidity risk factors to CDS premia only and of Chen, Fabozzi, and Sverdlove (2007) who calibrate a similar reduced-form model to CDS ask quotes or mid quotes only and deduce implied liquidity premia in bond prices. A counterintuitive estimation result of the latter study are the on average negative bond liquidity premia for all investment grade rating classes when CDS ask premia are used.

Third, we extend the empirical evidence of Nashikkar, Subrahmanyam, and Mahanti (2007) on the relation between the liquidity of the bond and the CDS market by disentangling the credit risk and liquidity premia. Instead of the absolute or relative CDS bid-ask spread which are affected by credit risk, our model allows us to determine pure and directly comparable liquidity premia for the bond and the CDS market. We obtain a significant relationship between these pure liquidity premia in excess of the liquidity spill-over from the bond to the CDS market which is immanent to our model. Specifically, we demonstrate that higher liquidity premia in the bond market lead to decreasing liquidity premia in the CDS market. The relation is particularly pronounced for the subinvestment grade sector, suggesting that the CDS market becomes a more attractive substitute for taking on credit risk synthetically when the bond market is illiquid.

The remainder of the paper is structured as follows. We introduce our reduced-form model in Section I and derive the credit risk, liquidity, and correlation premia in Section II. Section III presents the empirical results of the model calibration and a detailed analysis of the estimated time-varying premia. A stability analysis is provided in Section IV. Section V summarizes and concludes.

I. The Credit Risk and Liquidity Model

A. Specification of the Risk Structure

The first step in the model specification consists of the specification of the underlying risk structure. We assume a standard Duffie and Singleton (1997) framework in which default-free zero coupon bonds, default-
risky coupon-bearing bonds and CDS are traded. The liquidity of these instruments can differ, and we choose the default-free zero coupon bonds as “liquidity numéraire” with a liquidity discount factor equal to 1. We thus avoid specifying a perfectly liquid instrument in comparison to which each illiquid instrument trades at a discount.

The default-free term structure of interest rates is driven by one risk factor, the instantaneous default free interest rate process \( r(t) \). The credit risk for a specific bond issuer is characterized by a stochastic default-risk hazard rate \( \lambda(t) \), which is assumed to be reflected equally in CDS premia and corporate bond prices. The process \( \gamma^b(t) \) defines the liquidity intensity in the bond market. This process determines the fraction of a bond’s price due to liquidity deviations from the liquidity numéraire. In the CDS market, we use two liquidity intensities \( \gamma^{ask}(t) \) and \( \gamma^{bid}(t) \) to describe the individual liquidity effects for the CDS ask and bid premia. Conditional on the paths of these intensities,

\[
\tilde{D}(t, \tau) = \exp\left(-\int_t^\tau r(s) \, ds\right) \tag{1}
\]

is the discount factor for interest rates,

\[
\tilde{P}(t, \tau) = \exp\left(-\int_t^\tau \lambda(s) \, ds\right) \tag{2}
\]

equals the risk-neutral survival probability and

\[
\tilde{L}^l(t, \tau) = \exp\left(-\int_t^\tau \gamma^{l}(s) \, ds\right) \tag{3}
\]

is the liquidity discount factor in the bond market (\( l = b \)) and the CDS market (\( l = \text{ask, bid} \)).

We assume that \( r \) evolves independently from the default and liquidity intensities. The model can easily be generalized to capture correlation effects between \( r \) and the other risk factors. The default intensity \( \lambda \) and liquidity intensities for the bond (\( \gamma^b \)), the CDS ask premium (\( \gamma^{ask} \)), and the CDS bid premium (\( \gamma^{bid} \)) are determined by the four latent factors \( x, y^b, y^{ask}, \text{and } y^{bid} \). We model \( x \) as a square root process, \( y^b, y^{ask}, \text{and } y^{bid} \).
\( y_{\text{bid}} \) as arithmetic Brownian motions. The stochastic default and liquidity intensities are described by the following four-factor model:

\[
\begin{pmatrix}
    d\lambda(t) \\
    dy^b(t) \\
    dy^{ask}(t) \\
    dy^{bid}(t)
\end{pmatrix} = \begin{pmatrix}
    1 & g_b & g_{\text{ask}} & g_{\text{bid}} \\
    f_b & 1 & \omega_{b,\text{ask}} & \omega_{b,\text{bid}} \\
    f_{\text{ask}} & \omega_{b,\text{ask}} & 1 & \omega_{\text{ask},\text{bid}} \\
    f_{\text{bid}} & \omega_{b,\text{bid}} & \omega_{\text{ask},\text{bid}} & 1
\end{pmatrix} \begin{pmatrix}
    dx(t) \\
    dy^b(t) \\
    dy^{ask}(t) \\
    dy^{bid}(t)
\end{pmatrix}
\]

\[
(4)
\]

\[
\begin{pmatrix}
    \alpha - \beta x(t) \\
    \mu^b \\
    \mu^{\text{ask}} \\
    \mu^{\text{bid}}
\end{pmatrix} dt + \begin{pmatrix}
    \sigma \sqrt{x(t)} dW_x(t) \\
    \eta^b dW_{y^b}(t) \\
    \eta^{\text{ask}} dW_{y^{\text{ask}}}(t) \\
    \eta^{\text{bid}} dW_{y^{\text{bid}}}(t)
\end{pmatrix},
\]

\[
(5)
\]

with parameters \( \alpha, \beta, \mu^l, f_l, g_l, \sigma > 0 \), and \( \eta^l > 0 \). \( W_x \) and \( W_{y^l} \) are independent Brownian motions, \( l \in \{b, \text{ask}, \text{bid}\} \). The matrix of the factor sensitivities is assumed to be of full rank in order to ensure parameter identification.

\( f_l \) and \( g_l \) determine the correlation between \( \lambda \) and \( \gamma^l \). If both coefficients equal zero, credit risk and liquidity are uncorrelated. If \( f_l \neq 0 \), credit risk directly affects liquidity, and the reverse applies if \( g_l \neq 0 \).

There are two links that determine the correlation between the liquidity intensities. First, there can be an indirect link through the impact of \( x \) via the factor sensitivity \( f_l \). Second, the coefficients \( \omega_{l,k} \) imply a direct link between the liquidity intensities through the latent risk factors \( y_l \) and \( y_k \). Economically speaking, a correlation between the liquidity intensities which is not directly due to \( x \) allows us to determine the channel through which pure liquidity effects are transmitted from one market into the other. A potential relation between the CDS ask and bid liquidity intensities as measured by \( \omega_{\text{ask},\text{bid}} \) can be attributed to an inventory argument: If a trader enters into transactions on the ask side, thus taking on credit risk, she is likely to adjust the ask and bid premia accordingly in order to retain a balanced inventory, and vice versa. The bond liquidity intensity and the CDS ask and bid liquidity intensities, on the other hand, can be interdependent due to non-zero values of \( \omega_{b,\text{ask}} \) and \( \omega_{b,\text{bid}} \) because long (short) credit risk positions can be incurred either by buying (short-selling) the bond or by selling (buying) credit protection in the CDS contract on the ask (bid) side. A liquidity-driven price or premium change in one market presumably leads to corresponding changes in the other market: If the bond price falls due to a lower liquidity, buying credit risk in the bond market becomes cheaper. Therefore, a trader will obtain less transactions on the CDS bid side since investors have the opportunity to take on credit risk more cheaply directly in the bond market. The trader is then likely to
increase the bid premium in order to obtain trades on the bid side. The reverse effect applies for the CDS ask premium which we expect to decrease if the bond liquidity decreases. Due to the symmetric nature of these direct liquidity spillover effects, we choose a symmetric structure of the factor sensitivity matrix with regard to the $\omega_{l,k}$-coefficients.

B. Bond Market

We represent the value of a default-risky and potentially illiquid coupon-bearing bond as the expectation under a risk-neutral measure. If default occurs at time $\tau$, the bondholder recovers a fixed fraction $R$ of the face value $F$. Default can occur at any time, and recovery takes place on the first trading day following the default event. Hence, the time-$t$ price $CB(t)$ of a coupon-bearing bond with a fixed coupon $c$ paid at times $t_1, \ldots, t_n$, notional $F$, maturity in $t_n$, and recovery at times $\theta_j$ ($t \leq \theta_1 < \ldots < \theta_N \leq t_n$) is given by

$$CB(t) = c \cdot \sum_{i=1}^{n} D(t, t_i) E_t \left[ \bar{P}(t, t_i) \bar{L}^b(t, t_i) \right] + F \cdot D(t, t_n) E_t \left[ \bar{P}(t, t_n) \bar{L}^b(t, t_n) \right] + R \cdot F \cdot \sum_{j=1}^{N} D(t, \theta_j) E_t \left[ \Delta \bar{P}(t, \theta_j) \bar{L}^b(t, \theta_j) \right].$$

$E_t$ is the conditional expectation under the risk-neutral measure, and $\theta_0 := t$. Given that $r$ is independent of the other risk factors, we can compute $D(t, \tau) := E_t [D(t, \tau)]$ separately from the joint expectation of the default risk factor and the liquidity factor. $\Delta \bar{P}(t, \theta_j) := \bar{P}(t, \theta_{j-1}) - \bar{P}(t, \theta_j)$ denotes the probability of surviving from $t$ until $\theta_{j-1}$ and then defaulting between $\theta_{j-1}$ and $\theta_j$ conditional on the current date $t$.

Equation (6) can be interpreted as the expected present value of all future bond cash-flows: the first term gives the expected present value of the coupon payments at each coupon date. The second term equals the expected present value of the principal payment in the last period. The last term denotes the expected present value of the recovery rate payment. Therefore, the discount factor for each future payment consists of two terms: the risk-free discount factor $D$, and the joint expectation of the default risk factor $\bar{P}$ and the liquidity factor $\bar{L}$.

C. CDS Market

A CDS is a bilateral contract which allows two counterparties to trade the credit risk of an underlying reference obligation. In the fixed leg of the swap, the counterparty which buys credit protection agrees to
make periodic fee payments over the life of the swap. In return, the counterparty which sells credit protection agrees to make a payment if a “credit event” occurs for the reference obligations. This contingent payment is the floating leg of the swap.

In our model, we capture the following basic form of the CDS contract. At the inception, the protection buyer and seller agree on the CDS premium \( s \) which the buyer pays to the seller. The premia are quoted annualized and in basis points (bp) per unit of face value of the claim to which the credit protection applies. Premium payments are made in arrears on fixed payment dates; March, June, September, and December 20th have evolved as the standard dates. If a contract is entered into on a non-standard date, the time until the next standard date is added to the quoted maturity of the contract. In case of a credit event before the maturity of the CDS, the contract automatically terminates. The buyer pays the premium accrued since the last payment date to the seller, delivers the bond on which the CDS contract is written, and obtains the face value of the defaulted bond.

In practice, CDS contracts are written on multiple reference obligations, they can include multiple credit events, allow the protection buyer to choose from an entire delivery basket which asset to deliver upon default or to specify an auction process for cash settlement instead of physical delivery, and the payments are subject to counterparty risk. In our setting, we abstract from these features in order to keep the model tractable. The issuer simultaneously defaults on all bonds, and this immediately triggers the credit event. All bonds have the same post-default price, making the cheapest-to-deliver option worthless, and settlement occurs immediately upon default.

It is not obvious whether liquidity should be included in a model for CDS premia, and if so, in which way this should be done. After all, a CDS is a derivative, not an asset, and thus not exposed to illiquidity effects caused by a fixed supply or shorting costs such as bonds are. Both in empirical studies and in theoretical models, see e.g. Schueler and Galletto (2003) or Longstaff, Mithal, and Neis (2005), it is generally assumed that the CDS mid premium reflects a price which is entirely free of liquidity risk. Undoubtedly, however, the bid and ask premia reflect liquidity aspects of a CDS. From these two quotes, we will extract the unobservable, pure credit risk premium. Typically, this premium will differ from the mid premium. As a consequence, we model two values of the fixed leg of the CDS, one for the ask and one for the bid side.
The value of the fixed leg of a CDS contract at time \( t \) with fixed in-arrear premium payment \( s_{\text{ask}} \) at times \( T_1, \ldots, T_m \), maturity \( T_m \), and stochastic settlement times \( \theta_j \ (t \leq \theta_1 < \ldots < \theta_M \leq T_m) \) in case of a credit event equals

\[
CDS_{\text{fix}} (t) = s_{\text{ask}} \left( \sum_{i=1}^{m} D(t, T_i) E_t \left[ \tilde{P} (t, T_i-1) \tilde{L}_{\text{ask}} (t, T_i) \right] \right.
\]
\[
+ \sum_{j=1}^{M} D(t, \theta_j) \delta_j E_t \left[ \Delta \tilde{P} (t, \theta_j) L_{\text{ask}} (t, \theta_j) \right] \right).
\]

(7)

In equation (7), \( \delta_j \) accounts for the premium fraction accrued in the interval between the last premium payment and the settlement time \( \theta_j \). \( \tilde{L}_{\text{ask}} \) is defined as \( L^b \) with the bond liquidity intensity \( \gamma^b \) replaced by the CDS ask liquidity intensity \( \gamma^\text{ask} \). Equation (7) reflects that the payment of all ask premia \( s_{\text{ask}} \) has to be discounted for the default probability as the payment at time \( T_i-1 \) only occurs with a probability \( \tilde{P} (t, T_i-1) \). The CDS-specific liquidity discount factor for the ask premium \( L_{\text{ask}} (t, T_i) \) accounts for the possibility that part of the CDS ask premium is not due to default risk but to the fact that the protection seller demands an additional premium because of illiquidity.

The value of the floating leg, the expected discounted payment of the protection seller upon default is given by

\[
CDS_{\text{float}} (t) = F \sum_{j=1}^{M} D(t, \theta_j) E_t \left[ \Delta \tilde{P} (t, \theta_j) \right] - RD (t, \theta_j) E_t \left[ \Delta \tilde{P} (t, \theta_j) L^b (t, \theta_j) \right].
\]

(8)

The first term in equation (8) equals the discounted present value of the face value \( F \), the second equals the expected discounted present value of the defaulted bond which the protection seller obtains if she sells the delivered bond. Therefore, this second term is identical to the third term in the bond pricing equation (6) and thus contains the discounting factor for the bond liquidity in addition to the credit risk discounting factor. Economically speaking, the bond liquidity directly affects the floating leg of the CDS contract both in the case of physical delivery and cash settlement. A less liquid bond has a lower post-default price compared to an otherwise identical bond with higher liquidity. The CDS premium is therefore higher in order to compensate the protection seller for the lower value of the bond should default occur. This effect pertains even if the CDS market is perfectly liquid.
From equation (7) and (8) we obtain

\[ s^{\text{ask}}(t) = \frac{F \sum_i D(t, \theta_j) E_t \left[ (1 - R \tilde{L}^b(t, \theta_j)) \Delta \tilde{P}(t, \theta_j) \right]}{\sum_i D(t, T_i) E_t \left[ \tilde{P}(t, T_{i-1}) \tilde{L}^{\text{ask}}(t, T_i) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{\text{ask}}(t, \theta_j) \right]}, \]  

(9)

The closed-form solution for the CDS bid premium is identical to that for the ask premium with the only exception that \( \tilde{L}^{\text{ask}} \) is replaced by \( \tilde{L}^{\text{bid}} \).

\[ s^{\text{bid}}(t) = \frac{F \sum_i D(t, \theta_j) E_t \left[ (1 - R \tilde{L}^b(t, \theta_j)) \Delta \tilde{P}(t, \theta_j) \right]}{\sum_i D(t, T_i) E_t \left[ \tilde{P}(t, T_{i-1}) \tilde{L}^{\text{bid}}(t, T_i) \right] + \sum_j \delta_j D(t, \theta_j) E_t \left[ \Delta \tilde{P}(t, \theta_j) \tilde{L}^{\text{bid}}(t, \theta_j) \right]}, \]  

(10)

where equation (9) and (10) differ only with regard to the liquidity discount factor. Here, a short remark on the relative size of \( \tilde{L}^{\text{ask}} \) and \( \tilde{L}^{\text{bid}} \) is in order. The modeling of \( \gamma^{\text{ask}} \) and \( \gamma^{\text{bid}} \) in equation (4) does not guarantee that \( \tilde{L}^{\text{ask}} \leq \tilde{L}^{\text{bid}} \) and, therefore, \( s^{\text{ask}} \geq s^{\text{bid}} \). In our empirical study, however, we only obtain estimates that translate into this relationship.

Due to the structure of the factor model in equation (4) and the independence of \( x \) and \( y' \), the expected values of \( \tilde{P}(t, \tau_i) \cdot \tilde{L}^l(t, \tau_i) \) and \( \tilde{P}(t, \tau_i) \cdot \tilde{L}^l(t, \tau_{i+1}) \) in equation (6), (9), and (10) can be represented explicitly by analytical functions which result in an affine term-structure model. These analytical representations are derived explicitly in the appendix. Substituting these functions in equation (6), (9), and (10) yields the analytical solutions for the bond price \( CB(t) = CB(t, x, y; f, g) \), for the CDS ask premium \( s^{\text{ask}}(t) = s^{\text{ask}}(t, x, y; f, g) \), and for the CDS bid premium \( s^{\text{bid}}(t) = s^{\text{bid}}(t, x, y; f, g) \), where \( y = (y^b, y^{\text{ask}}, y^{\text{bid}}), f = (f_b, f_{\text{ask}}, f_{\text{bid}}), \text{ and } g = (g_b, g_{\text{ask}}, g_{\text{bid}}) \).

**II. Measures for Credit Risk, Liquidity, and Correlation Premia**

The model developed in Section I allows us to disentangle the total bond spread \( bs \) into a pure default risk component \( bd \), a pure liquidity component \( bl \), and a correlation-induced component \( bc \). By an analogous procedure based on bid and ask quotes of CDS premia, we can compute a pure credit risk component \( sd \), a pure liquidity component \( sl \), and a correlation-induced component \( sc \). The rationale for this decomposition is most obvious for the bond. The credit risk premium \( bd \) equals the bond spread that would apply if credit risk were the only priced factor (excepting, of course, \( r \)). In this case, the latent factor \( y^b \) is identical to 0, the factor sensitivities \( f \) and \( g \) become irrelevant, and the credit risk intensity \( \lambda \) and the latent factor \( x \)
coincide. The liquidity premium \( bl \) equals the bond spread that would apply if liquidity were the only priced factor, i.e., \( x \) is identical to 0, and the latent factor \( y^b \) and the liquidity intensity \( \gamma^b \) coincide. The correlation premium \( bc \) then measures the additional bond spread that is incurred because the credit risk and liquidity intensities \( \lambda \) and \( \gamma^b \) are correlated.

Assuming a perfectly liquid bond and CDS market, the bond spread is directly comparable to the CDS premium if the maturity of both instruments is identical and if, in addition, the bond price equals its face value. The second condition is important to avoid the difficulties discussed by Duffie (1999) and Duffie and Liu (2001) who show that the yield spreads on fixed-coupon corporate bonds cannot be directly compared to CDS premia. Therefore, we define a bond’s pure credit risk premium \( bd \) for a given value of \( x \) in two steps. First, we assume that \( y \) and the factor sensitivities \( f \) and \( g \) equal 0 and determine the coupon \( c_{\text{par}} \) that makes the theoretical bond price in equation (6) equal to par, i.e., \( CB(x,0,t;0,0) \) equals \( F \) for \( c_{\text{par}} \). Second, we compute \( bd \) as the bond spread over the risk-free rate for this bond:

\[
CB(x,0,t;0,0) = \sum_{i=1}^{m} \frac{c_{\text{par}}}{(1+y(t,T_i) + bd)(T_i-t)} + \frac{F}{(1+y(t,T_m) + bd)(T_m-t)}
\]

(11)

where \( y(t,T_i) = D(t,T_i)^{-\frac{1}{T_i-t}} - 1 \) equals the yield-to-maturity of a default-free zero bond with reference liquidity and maturity \( T_i-t \).

The bond liquidity premium \( bl \) follows as the premium increase in excess of \( bd \) if the impact of the latent liquidity factors \( y \) is included but the correlation between the credit risk and liquidity intensities equals 0, i.e., the bond spread increase for \( CB(t,x,y;0,0) \). The correlation premium \( bc \) then arises naturally as the difference between the total bond spread \( bs \) for the bond price \( CB(t,x,y,f,g) \) which includes the non-zero factor sensitivities \( f \) and \( g \), and the sum of the credit risk and the correlation premia, \( bd + bl \).

We define the credit risk, liquidity, and correlation components of a CDS analogously to the procedure in the bond market. First, we compute the pure credit risk premium \( sd \) by assuming that the liquidity discount factors \( L^\text{ask} \) or \( L^\text{bid} \) are equal to 1. Equation (9) and (10) illustrate that in this case, \( sd \) is exclusively determined by the default-free interest rates, the default probability, and the bond liquidity.

In a CDS market whose liquidity differs from the liquidity numéraire, the ask and bid premia differ from the pure credit risk premium \( sd \). In line with the literature on market microstructure, it seems apparent to select the size of the bid-ask-spread as a measure of illiquidity. This is not an appropriate approach in our
context for two reasons. First, a comparison of (9) and (10) shows that the bid-ask-spread is also affected by pure credit risk. Assume that only the latent credit risk factor $x$ and thus the default intensity increases, then the ask premium increases more strongly than the bid premium does. Second, the bid-ask-spread, even if taken relatively to $sd$, is not comparable to our liquidity measure $bl$ in the bond market.

We therefore proceed analogously to the bond market and define the liquidity premium in the CDS market $sl$ by

$$sl = \frac{1}{2} (s^{\text{ask}}(t,x,y;0,0) + s^{\text{bid}}(t,x,y;0,0)) - sd,$$

i.e., $sl$ is the difference between the theoretical mid premium for uncorrelated credit risk and liquidity intensities and the pure credit risk premium $sd$. This definition of $sl$ corresponds fully to the definition of the bond liquidity premium $bl$. In addition to this formal analogy, $sl$ allows for an inventory-related interpretation: If a trader has entered into a number of CDS contracts as protection seller, she moves the ask premium and the bid premium at which she is willing to trade upwards in order to balance her inventory. Since the pure credit risk premium $sd$ remains at its initial value while $s^{\text{ask}}$ and $s^{\text{bid}}$ increase, $sl$ increases as well. If, on the other hand, demand for transactions on the bid side increases and the trader ends up with an increased short credit risk position, she will set lower bid and ask quotes in order to cancel out this inventory imbalance. This results in lower values of $sl$.

Our measure of CDS liquidity is thus consistent with the measure of the bond liquidity premia: if a large number of investors want to sell credit risk by selling bonds — which can be interpreted as buying credit protection — the liquidity premium in the bond market increases and vice versa.

Finally, the CDS correlation premium $sc$ equals the difference between the mid premium that includes the impact of the factor sensitivities $f$ and $g$ and the theoretical, correlation-free mid premium.

### III. Empirical Analysis

#### A. Data

We exclusively focus on data from the Euro area since the sample of Euro-denominated CDS contracts is much larger than that of US-Dollar denominated contracts in the early phase of our research interval: Between June 1st, 2001 and September 30th, 2001, we observe CDS ask and bid quotes on 119 Euro-
denominated contracts in contrast to 16 US-Dollar denominated contracts. For the current term structure of the default-free interest rates, we use the estimates which are provided by the Deutsche Bundesbank on a daily basis. These estimates are determined by means of the Nelson-Siegel-Svensson method from prices of German Government Bonds which represent the benchmark bonds in the Euro area for most maturities.\footnote{1} From this term structure of interest rates, we compute prices of default-free zero-coupon bonds which we assume to have the reference liquidity discount factor of 1. The recovery rate is assumed to equal 40%.

All CDS and bond data is collected via the Bloomberg system. The daily CDS ask and bid closing premia for the senior unsecured debt class were made available to us by an international investment bank. We compute the daily closing mid premia for CDS to compare them to the bonds’ yield spreads which are derived from Bloomberg mid yields. The research period runs from June 1, 2001 to June 30, 2007. This period covers 1,548 trading days. We restrict ourselves to using CDS premia with a reference maturity of 5 years to obtain a sample with homogenous CDS liquidity. According to the time conventions in the CDS market described in Section I.C, we obtain the true CDS maturities by adding the distance between the quoting day and the next reference date to the quoted maturity of 5 years.

Bond data are also obtained from Bloomberg. We collect all bond mid prices for firms which had at least 2 bonds outstanding at some point-in-time during the observation interval. Furthermore, we drop all firms with fewer than 20 consecutive trading days on which at least two bond prices as well as the bid and ask CDS premium were available.

For each of the remaining firms, we collect the rating history from Bloomberg for the period during which we observe bond prices and CDS premia. Both the Standard&Poor’s (S&P) rating and the Moody’s rating are used and mapped on a numerical scale ranging from 1 to 50 in which 1 corresponds to an “AAA” S&P rating (“Aaa” Moody’s rating). The highest value, 50, corresponds to a “CCC+” S&P rating (“Caa1” Moody’s rating) and is the lowest rating which we observe during the observation interval. If the resulting numerical rating of S&P and Moody’s differs by 2 or more, we take the average of the two ratings. If the rating differs by 1, we choose the more conservative rating. If no rating can be found for at least 20 observations on consecutive trading days during which at least two bond prices as well as the bid and ask CDS premium were available, we drop the firm from our sample.

The above procedure leaves us with a set of 155 firms from 8 corporate sectors and a numerical rating history that consistently lies between 1 and 50. A detailed overview is given in Table I.
For ease of exposition, we first compute the average numerical rating of a firm for all days during which there are a sufficient number of observations. We then map the numerical value to the S&P rating and use this as the column heading. Table I shows that the majority of firms has a time-series average rating in the investment grade sector; only 9 lie in the subinvestment grade range. The largest industry group is the sector “Financials” with a total of 54 firms which are also among the top-rated ones. Overall, Table I demonstrates that our sample is skewed towards financial firms and firms in the investment grade sector.

To present the time-series of bond yield spreads and CDS premia, we compute the average bond spread and CDS mid premium for each rating class at every date of our observation interval as follows. First, we identify the rating for a particular firm on each day. We then compute the bond spread for each bond of that particular firm as the difference between its yield and that of a synthetical default-free bond with identical coupon and maturity. Next, we interpolate the resulting yield spreads to obtain a maturity of 5 years. We proceed by taking averages of the obtained yield spreads and the observed CDS mid premia for all firms with an average investment, respectively subinvestment grade rating. The resulting time series for the investment grade and subinvestment grade are depicted in Figure 1.

As we see from Figure 1, the mean investment grade bond yield spreads consistently exceed the mean mid CDS premia. Overall, the mean investment grade bond spread has a time-series average of 89.42 bp with a time-series standard deviation of 23.03 bp, fluctuating between 33.54 bp and 178.86 bp. The mean investment grade CDS premia fluctuate between 15.86 bp and 143.76 bp with a time-series average of 45.42 bp and a standard deviation of 27.96 bp. The difference between the two time series gives a first impression of the liquidity-induced differences between the bond and the CDS market, but the coupon effect and the simple interpolation scheme only allow a rough comparison.

The mean yield spreads for the subinvestment grade sector have an average of 369.62 bp and a time-series standard deviation of 188.70 bp. Overall, the mean subinvestment grade bond spread fluctuates between 87.60 bp and 1,320.33 bp. The mean subinvestment grade CDS mid premia are partly above and partly below the bond spreads. Their time-series average of 341.33 bp lies below, their standard deviation of 224.53 bp above the corresponding values of the bond spread.
B. Estimation Procedure

We estimate the parameters and the current factor values of the 4 intensity processes individually for each of the 155 firms from the observed senior unsecured bond prices and CDS premia. In total, we estimate for each firm the 9 parameters \((\alpha, \beta, \sigma, \mu^b, \eta^b, \mu^{ask}, \eta^{ask}, \mu^{bid}, \eta^{bid})\), the 9 factor sensitivities \(f = (f_b, f_{ask}, f_{bid})\), \(g = (g_b, g_{ask}, g_{bid})\), and \(\omega = (\omega_{b, ask}, \omega_{b, bid}, \omega_{ask, bid})\), and for each date \(t\) the current value of the intensities \((\lambda, \gamma^b, \gamma^{ask}, \gamma^{bid})(t), t = 1, \ldots, 1548.2\).

The estimation procedure consists of three basic steps. In the first step, we initiate a base grid for the process parameters \((\alpha, \beta, \sigma, \mu^b, \eta^b, \mu^{ask}, \eta^{ask}, \mu^{bid}, \eta^{bid})\), and set all factor sensitivities \(f, g, \omega\) to 0. This corresponds to the case of uncorrelated intensities. In the second step, we then determine the values \((\lambda, \gamma^b, \gamma^{ask}, \gamma^{bid})(t), t = 1, \ldots, 1548\), which simultaneously minimize the sum of squared errors between the time series of the observed and the theoretical CDS premia and bond yield spreads. This second step matches all values at the basis point level, and estimation is conditional on the presumed process parameters and factor sensitivities. In the third step, we determine the factor sensitivities \(f, g, \omega\) which are implied by the estimated time series of the intensities using a discrete version of equation (4). We iterate between the second and the third step using the updated factor sensitivities and intensity values until we obtain no further absolute change larger than 0.01 in the factor sensitivities in two subsequent steps.\(^3\) We follow this procedure in each grid point and determine the point associated with the smallest sum of squared errors. Around this point, we initiate a finer local grid as in the first step and repeat the second and the third step in each point of the new grid. We stop this three-step estimation procedure when the minimal sum of squared errors twice decreases by less than 1% on two subsequent grid specifications. In order to control for local optima, we repeat the analysis for the points in the base grid associated with the second and third smallest sum of squared errors.

Having thus determined the estimates of the process parameters, the intensities and the factor sensitivities, we subsequently compute the credit risk, liquidity, and correlation premia for bonds and CDS as explained in Section II.
C. Credit Risk, Liquidity, and Correlation Premia: Cross-Sectional Results

C.1. Factor Sensitivities

We first discuss the coefficient estimates for the factor matrix in equation (4). This allows us to demonstrate how credit risk affects liquidity, how liquidity affects credit risk, and how the liquidity of the bond and the CDS market affect one another.

Insert Table II about here.

As the estimates for the factor sensitivities in Table II show, credit risk has an impact on both the bond liquidity intensity and the CDS liquidity intensities but not vice versa. The latent factor \( x \) affects the bond liquidity intensity \( \gamma^b \) significantly for 140 out of 155 firms. 138 of these estimates for \( f_b \) are positive, and the 2 negative estimates are obtained for one utility and one financial firm which have an AAA, respectively an AA, rating. The positive mean factor sensitivity estimate of 0.16 suggests that the liquidity of the bond market dries up as credit risk increases; we quantify the impact on the premia components in more detail below. The impact of \( x \) on the CDS ask intensity \( \gamma^\text{ask} \), measured by \( f_{\text{ask}} \), is significant for 138 and positive for 137 firms with a mean estimate of 0.37. The CDS bid intensity \( \gamma^\text{bid} \), in turn, is significantly affected by \( x \) for only 66 firms with a negative estimate for \( f_{\text{bid}} \) for 37 firms. The mean estimate of -0.07 is, however, significantly different from 0 at the 1% level and implies that the CDS bid quotes decrease disproportionately when credit risk increases.

The impact of the latent factors \( y^b, y^\text{ask}, \) and \( y^\text{bid} \) on the default intensity \( \lambda \), on the other hand, is almost negligible: we obtain only one significant coefficient estimate for \( g_b \), three for \( g_{\text{ask}} \) – out of which two are positive – and two for \( g_{\text{bid}} \) with a positive and a negative one. These results illustrate that credit risk premia increase liquidity premia in the bond market but not vice versa. We can also conclude that higher credit risk leads to a higher distance between the pure credit risk CDS premium and the ask premium. CDS bid premia, on the other hand, are not as unilaterally affected.

The liquidity spillover between the bond and the CDS market can be inferred from the estimates of \( \omega_{b,\text{ask}} \) and \( \omega_{b,\text{bid}} \). The coefficient estimate for \( \omega_{b,\text{ask}} \) is significant for 123 firms and negative for 118. The mean value of -0.02 implies that a decreasing liquidity in the bond market results in lower CDS ask premia. This is consistent with a substitution effect in the bond and the CDS market. A decreasing liquidity in the bond market
market implies that buying credit risk through the bond becomes cheaper due to decreasing bond prices and increasing bond spreads, and thus more attractive, while short credit risk positions becomes less attractive. Therefore, investors who intend to go short credit risk are less likely to buy credit risk protection via an ask-induced CDS trade. Therefore, the trader has to decrease her ask quotes in order to obtain transactions on the ask side. Going long credit risk via a CDS, on the other hand, becomes less attractive for investors when the bond liquidity decreases. In order to obtain transactions on the bid side, the trader must therefore increase her bid quote compared to the case with high bond market liquidity. The estimate for the CDS bid liquidity coefficient $\omega_{b,bid}$ which is significant for 85 firms and positive for 80 with a mean value of 0.01 is consistent with this substitution of bonds and CDS. The estimate for $\omega_{ask,bid}$ is significant for 131 firms and positive for 116 firms. The negative mean of -0.38 implies that the bid and ask quote tend to move in opposite directions. This finding agrees with an overall increasing liquidity in the CDS market with decreasing bid-ask spreads as the market matures.

Comparing the results for the investment and the subinvestment grade sector, we observe a similar result as for the entire sample. Only the absolute value of the coefficient estimates tends to be larger in the subinvestment grade sector which points to a stronger relation between the bond and the CDS market than in the investment grade sector. We will further explore this result in Section III.D.

C.2. Premia Components

We now analyze the components of the bond spread and the CDS premium as they are disentangled by our model. Table III displays the results.

Table III demonstrates that the credit risk, liquidity, and correlation premia increase as the rating deteriorates. As to the AAA rating class, the pure credit risk premium in yield spreads $bd$ has an average of 6.11 bp which approximately doubles for each rating downgrade in the investment grade sector. The subinvestment grade sector exhibits values of $bd$ which are at least five times as large.

Concerning the liquidity premia $bl$, the increase from the investment to the subinvestment grade sector is less steep, although we still obtain a strictly positive estimate for $bl$ for each firm. In addition, the minimum and maximum of $bl$ do not monotonously increase for a decreasing rating. The average correlation premia
bc increase in the rating up to the CCC rating class and are strictly positive except for the AAA rating class. This negative average rating is driven by the negative estimates of the parameter $f_b$. On average, $bd$ accounts for 60% of the total bond spread, $bl$ for 35%, and $bc$ for 5%. These results are in sharp contrast to the estimated default components in the studies by Elton, Gruber, Agrawal, and Mann (2001) and Huang and Huang (2003) who report that the non-default component accounts for the largest percentage of the bond spread.

The CDS pure credit risk premia $sd$ consistently exceed $bd$ by a relatively small amount. This exceedance is due to the model-immanent liquidity spillover from the bond to the CDS market if a default occurs. The minimal difference between $bd$ and $sd$ is attained for the AAA rating class with on average 0.07 bp and the maximal one for the B class with on average 4.97 bp. This is consistent with the increasing average level of the bond liquidity premia $bl$.

The final results of Table III concern the CDS pure liquidity premia $sl$ and the correlation premia $sc$. As explained in Section II, we measure the liquidity of the CDS market by the asymmetry between the ask and the bid quotes relative to the pure credit risk premium: If our estimate of $sd$ is closer to the bid than to the ask quote, $sl$ has a positive value and vice versa. On average, the liquidity premium $sl$ is positive, which, as argued in Section II, suggests that transactions in the CDS market are mainly ask-initiated. The asymmetry increases and results in higher liquidity premia as the rating deteriorates. However, relative to the pure credit risk premia, the pure liquidity premia are smaller for the subinvestment grade sector, and 19.15% of the CDS liquidity premia are in effect negative. This result shows that the distribution of the liquidity premia is more symmetric than for investment grade CDS. In the next section, we attribute the negative liquidity premia to unusual market events. As for the bond market, the relative liquidity premia decrease in a particularly pronounced way for the transition from the investment grade to the subinvestment grade sector. In contrast to the bond market, on the other hand, we find that the CDS liquidity premia are much smaller for all rating classes. Their average size across all rating classes is only 1.94 bp compared to 26.36 bp in the bond market.

The average of the correlation premium $sc$ is almost negligible for the investment grade sector and grows by more than a factor of 10 for the subinvestment grade sector. This observation suggests that changes in credit risk result in a stronger decline of liquidity in the subinvestment than in the investment grade sector. We will explore the dynamic relation between the pure credit risk, the pure liquidity, and the correlation premia in greater detail in the following section. The negative minima of $sc$ are due to the fact that for some firms, the sensitivity of the CDS bid liquidity intensity to the latent credit risk factor is larger than that of the
CDS ask liquidity intensity. Therefore, the CDS bid premium can at times increase more strongly than the ask premium. Concerning the decomposition of the total CDS premia, we observe that on average 95% of the total premium is due to \( sd \), 4% to \( sl \), and 1% to \( sc \).

D. Credit Risk, Liquidity, and Correlation Premia: Time-Series Results

D.1. Comparison of Bond Yield Spread and CDS Premia Components over Time

The estimated credit risk, liquidity, and correlation premia components are depicted in Figure 2. For ease of presentation, only the averages of the investment and subinvestment grade premia are presented.

Insert Figure 2 about here.

Panels A and B of Figure 2 show that the pure credit risk premia both in yield spreads and CDS premia are almost identical both for the investment grade and the subinvestment grade sector. For the investment grade sector, there are two distinct spikes in late 2001 and late 2002 at the Enron and WorldCom defaults. The reaction of the subinvestment grade sector to the Enron default is almost negligible which may be due to the fact that our sample only consists of 2 subinvestment grade firms between June 2001 and February 2002. Overall, we observe the well-known decline of the pure credit risk premia time series \( bd \) and \( sd \). The end of the observation interval coincides with the beginning of the subprime crisis.

The bond liquidity premia \( bl \) exhibit a different behavior across the investment and subinvestment grade sectors as we observe from Panel A of Figure 2. During the high credit risk periods, the liquidity premia are volatile and flatten out at a higher level during the latter part of the observation interval for the investment grade sector. In the subinvestment grade sector, \( bl \) is highest shortly after the high-risk periods and decreases to a lower level towards the end of the observation interval. Visual inspection of the CDS-specific liquidity premia \( sl \) in Panel B of Figure 2 is more difficult since the absolute values are small. For both rating sectors, we observe a trend towards 0 as the CDS market matures. Overall, the level of \( sl \) is close to 0 for the entire observation interval in the investment grade sector, but liquidity premia are higher when credit risk is high. In the subinvestment grade sector, \( sl \) strongly fluctuates and becomes mostly negative when credit risk is high. This finding suggests that the ask-initiated transactions are partly replaced by bid-initiated transactions for the subinvestment grade sector, pointing at a high number of investors who attempt to take on credit risk synthetically in the CDS market.
Due to the insignificant estimates for the factor sensitivity of the credit risk intensity \( \lambda \) to the liquidity risk factor \( y \), the correlation premia \( bc \) and \( sc \) are closely associated with the credit risk premia. On comparing \( bl \) and \( bc \), Panel A of Figure 2 shows that the pure liquidity premia lie below the correlation premia during high-risk periods and above during low-risk periods for the investment grade sector. In the subinvestment grade sector, we observe a similar result during high risk periods, e.g. at the WorldCom default in late 2002. Overall, however, \( bl \) tends to be higher than \( bc \) in the lower rating classes. We interpret this as an indication that liquidity may dry up disproportionately in high credit risk phases, in particular for the investment grade sector. This agrees with the flight to quality and the flight to liquidity effects which are theoretically derived by Vayanos (2004) and documented empirically by Beber, Brandt, and Kavajecz (2007). The CDS correlation premia are, in contrast, almost negligible, i.e. the CDS liquidity is mostly independent of credit risk.

**D.2. Dynamic Interaction between the Bond and the CDS Market**

In order to study the dynamic interaction between the bond and the CDS market, we perform a time-series analysis of the premia across the different markets. Since the credit risk premia for both markets are computed with regard to the identical default intensity, \( bd \) and \( sd \) contain identical information except for the effect of bond liquidity on \( sd \). The same holds true for \( bd \) and \( bc \) since the estimates of the impact of the bond liquidity factor \( y_b \) on the default intensity \( \lambda \), \( g^b \), do not differ significantly from zero for almost all firms. Therefore, we focus on the pairwise relation between the credit risk, the liquidity, and the correlation premia across the two markets.
We use a vector error correction model (VECM) to study the long-run equilibrium relationship between the premia in the different markets and the reactions to short-run deviations. The specifications which we use are of the standard Johansen form:

\[
\begin{pmatrix}
\Delta bd_t \\
\Delta sd_t
\end{pmatrix} = \begin{pmatrix} 1 & \beta_d \end{pmatrix} \begin{pmatrix}
bd_{t-1} \\
\Delta sd_{t-1}
\end{pmatrix} + \sum_{j=1}^{5} \Gamma_{d,j} \begin{pmatrix} \Delta bd_{t-j} \\
\Delta sd_{t-j}
\end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\
\epsilon_{2,t}
\end{pmatrix},
\]

(13)

\[
\begin{pmatrix}
\Delta bl_t \\
\Delta sl_t
\end{pmatrix} = \begin{pmatrix} 1 & \beta_l \end{pmatrix} \begin{pmatrix}
bl_{t-1} \\
\Delta sl_{t-1}
\end{pmatrix} + \sum_{j=1}^{5} \Gamma_{l,j} \begin{pmatrix} \Delta bl_{t-j} \\
\Delta sl_{t-j}
\end{pmatrix} + \begin{pmatrix} \epsilon_{3,t} \\
\epsilon_{4,t}
\end{pmatrix},
\]

(14)

\[
\begin{pmatrix}
\Delta bc_t \\
\Delta sc_t
\end{pmatrix} = \begin{pmatrix} 1 & \beta_c \end{pmatrix} \begin{pmatrix}
bc_{t-1} \\
\Delta sc_{t-1}
\end{pmatrix} + \sum_{j=1}^{5} \Gamma_{c,j} \begin{pmatrix} \Delta bc_{t-j} \\
\Delta sc_{t-j}
\end{pmatrix} + \begin{pmatrix} \epsilon_{5,t} \\
\epsilon_{6,t}
\end{pmatrix},
\]

(15)

where \(\alpha_k = (\alpha_{k,b}, \alpha_{k,s})^\top\) is the error correction term’s impact coefficient for the bond spread \((b)\) and the CDS premium component \((s)\), \(\beta_k\) is the cointegration coefficient, and \(\Gamma_{k,j} = \begin{pmatrix} \Gamma_{b,b} & \Gamma_{b,s} \\
\Gamma_{s,b} & \Gamma_{s,s}
\end{pmatrix}_{k,j}\) is the \(2 \times 2\) coefficient matrix for the premia differences with lag \(j\), \(k \in \{d, l, c\}\). Time lags up to 5 trading days are considered to capture a weekly time interval. The resulting parameter estimates are transformed into a single estimate. We estimate the parameters separately for all firms. The results of the estimation are displayed in Table IV.

Insert Table IV about here.

As the estimated coefficients in Panel A of Table IV indicate, \(bd\) and \(sd\) are cointegrated with \(\beta_d = -1\). This value suggests that there is an almost perfect one-to-one relation between the two credit risk premia and that the effect of the bond liquidity on \(sd\) is almost negligible. The estimates of the error correction terms \(\alpha_{d,b} = -0.89\) and \(\alpha_{d,s} = -0.19\) imply that both the bond and the CDS premia react to deviations from this one-to-one relation and that the bond premia are more sensitive. The coefficient estimates for the lagged premia changes show that the changes are negatively autocorrelated and positively cross-autocorrelated with the negative autocorrelation dominating. The adjusted \(R^2\) of 15.35\% for \(\Delta bd\) and 13.22\% for \(\Delta sd\) is rather low, suggesting that premium changes are mostly caused by sudden changes in the firm’s creditworthiness.

The mean estimated cointegration coefficient \(\beta_l\) for the bond and CDS liquidity premia \(bl\) and \(sl\) equals 9.38 for the entire sample which implies that bond and CDS premia move in opposite directions over time.
The error correction term for $\Delta bl$, $\alpha_{t,b} = -0.05$, is only significant at the 10% level while the error correction term for $\Delta sl$, $\alpha_{t,s} = -0.15$ is significant at the 1% level. The autocorrelation of the premia changes is negative with an absolutely larger coefficient estimate for $\Gamma_{b,b} = -0.32$ than for $\Gamma_{s,s} = -0.22$. A central result is that the cross-market impact of the liquidity premium changes is one-sided: $\Delta bl_{t-1}$ significantly affects $\Delta sl_t$ with a coefficient estimate of $\Gamma_{b,s} = -0.02$ but the estimate of $\Gamma_{s,b}$ does not differ significantly from 0. Due to the overall lower level of $sl$, the impact of $\Delta bl_{t-1}$ is also economically significant. This allows the interpretation that a decrease of liquidity in the bond market makes it relatively more attractive to take on credit risk by selling protection in the CDS market. Therefore the CDS ask quotes decrease, the mid CDS premium moves towards the pure credit risk premium, and $sl$ decreases. Reversely, the weakly significant estimate of $\alpha_{t,b}$ and the insignificant estimate for $\Gamma_{s,b}$ imply that the CDS market’s liquidity does not affect the bond market’s liquidity. Consequently, the adjusted $R^2$ for $\Delta bl$ of 26.65% is lower than the adjusted $R^2$ for $\Delta sl$ of 33.05%, suggesting a significant interdependence between the markets’ liquidity.

The bond and CDS correlation premia $bc$ and $sc$ exhibit a negative estimate of the cointegration coefficient $\beta_c$ which points at a comovement of the credit-risk related part of the liquidity premia in both markets. With a value of -62.92, the estimate is rather large, and we attribute the size of the estimate to the relation in the investment grade sector below. The time-series behavior of $\Delta bc$ and $\Delta sc$ resembles that of the credit risk premia with negative autocorrelation and positive cross-autocorrelation. For $bc$, this similarity comes from the fact that the latent credit risk factor $x$ affects the bond liquidity intensity $\gamma^b$ but the default intensity $\lambda$ is unaffected by the latent bond liquidity factor $y^b$. For the CDS correlation premium $sc$, on the other hand, the similarity to $sd$ is less pronounced which is due to the simultaneous impact of $x$ on the bid and the ask liquidity intensity $\gamma^{bid}$ and $\gamma^{ask}$. The adjusted $R^2$ reflects this time-series behavior as well: for $\Delta bc$, the adjusted $R^2$ of 10.03% is in a similar range as for $\Delta bd$, while the value of 3.12% for $\Delta sc$ is much smaller. Here, a short remark on the effect of separating the total liquidity premia into a pure liquidity and correlation-induced component is in order. If we were to model the intensities independently, $bc$ would be subsumed in $bl$ and $sc$ in $sl$ since the liquidity intensity is unaffected by the liquidity factors $y^b$, $y^{ask}$, and $y^{bid}$. This is important because of its implications for the dynamics of the liquidity premia. As Table IV shows, the pure liquidity premia move in opposite directions while the correlation premia exhibit a high degree of co-movement with a much larger cointegration coefficient. Therefore, subsuming the pure liquidity component and the credit-risk induced correlation component would obscure the actual countermovement of $bl$ and $sl$. 
As discussed above, the time-series behavior of the premia partly differs between the investment and the subinvestment grade sector. The results of the VECM analysis for the investment grade sector in Panel B of Table IV show that the dynamics of the premia are the same as in Panel A but that the size of the coefficient estimates and the explanatory power decrease. For $\Delta bd$ and $\Delta sd$, only the estimate for $\beta_d$ remains unchanged at -1.00, but the error correction term coefficients, the autocorrelation coefficients, and the cross-autocorrelation coefficients exhibit an absolute decrease as does the adjusted $R^2$. Economically, this finding implies that deviations from the one-to-one relation between the credit risk premia are more persistent in the investment grade sector. The liquidity premia exhibit a very similar behavior in the investment grade sector as for the entire sample; the estimates for the cointegration coefficient, the error correction terms, and the cross-autocorrelation decrease very slightly. The autocorrelation terms, on the other hand, exhibit a slight increase for $\Delta bl$ and almost double for $\Delta sl$. As a result, the adjusted $R^2$ increases to 27.25% for $\Delta bl$ and 37.88% for $\Delta sl$. The correlation premia show an almost identical behavior in the investment grade sector as they do for the entire sample. The estimate for the cointegration coefficient $\beta_c$ decreases to -70.52, explaining the large coefficient estimate for the entire sample. Overall, the investment grade sector exhibits a slightly lower connection between the premia in the bond and the CDS market. These findings suggest that the premia for investment grade firms may be affected by market-specific conditions in excess of the firm-specific ones.

Panel C of Table IV shows the results for the subinvestment grade sector. For the credit risk premia, the estimate for the cointegration coefficient $\beta_d = -0.98$ is slightly lower than in the investment grade sector which we attribute to the additional effect of bond liquidity on $sd$. The estimate of the error correction terms, on the other hand, are significantly higher than for the investment grade sector as is the adjusted $R^2$. Clearly, $bd$ and $sd$ react more rapidly to deviations from their one-to-one relation than in the investment grade sector, i.e. the effect of the bond liquidity on $sd$ mainly consists of a level shift. The coefficient estimate for $\beta_l$ increases to 17.29 and the estimate for $\alpha_d$ decreases to -0.26, suggesting that the CDS market becomes more sensitive to the bond market’s liquidity. Due to the countermovement of the liquidity premia, taking on credit risk by selling protection in a CDS contract becomes a more attractive substitute to buying the bond than in the investment grade sector. The correlation premia exhibit an absolutely lower cointegration coefficient estimate with $\beta_c = -5.96$. This agrees with the size of the correlation premia which are closer than in the investment grade sector. Overall, the coefficient estimates imply that the bond and the CDS market in the subinvestment grade sector are more closely interconnected than in the investment grade sector.
IV. Stability of Credit Risk, Liquidity, and Correlation Premia

In this section, we perform a stability analysis of our results for the estimated credit risk, liquidity, and correlation components in bond yield spreads and CDS premia. We first explore the effect of ignoring liquidity in the CDS market on the bond spread decomposition. We subsequently analyze whether the premia dynamics identified in section III.D.2 are robust against the inclusion of exogenous variables and in phases of increasing and decreasing market-wide credit risk. We finally compare the premia which are obtained if the default and liquidity intensities are estimated from CDS ask or bid quotes only to the estimate which uses both observations.

A. The Effect of Excluding CDS Illiquidity

First, we explore whether the strictly positive bond liquidity premia $bl$ are a result of including stochastic liquidity in CDS ask and bid premia or simply a property of our data set. To do so, we propose the following modification of our model: first, we shift our focus to the CDS mid premia in our estimation procedure since there is no theoretically compelling reason why $sd$ must differ systematically from the mid premium. Second, we re-estimate the default and bond liquidity intensity time-series under the restriction $\gamma^l = \mu^l = \eta^l = 0$, $l = \text{ask, bid}$. This is basically the approach by Longstaff, Mithal, and Neis (2005) and suggests that the CDS market is the liquidity numéraire. Lastly, we compute the bond pure credit risk, pure liquidity, and correlation premia $bd$, $bl$, and $bc$ and compare them to the results from the initial estimation which includes liquidity in CDS premia. Since the effect only pertains when bonds are liquid relative to the CDS, we first present the estimated time-series for a single firm, the Dutch communications company The Nielsen Company (formerly VNU Group B.V.). The results are displayed in Figure 3.

Insert Figure 3 about here.

As we see in Figure 3, the estimated credit risk premium $bd$ has similar dynamics, regardless of whether CDS liquidity is included or not, but there are also clear differences. Overall, when stochastic CDS liquidity is excluded, $bd$ is higher, fluctuating between 33.23 bp and 510.83 bp with a mean of 120.67 bp and a standard deviation of 92.68 bp. When stochastic CDS liquidity is included, $bd$ lies between 32.88 bp and 394.95 bp, the mean equals 97.42 bp, and the standard deviation 67.51 bp. The differences are largest during the beginning of our observation interval when the CDS market was less liquid.
Conversely, the bond liquidity premium $bl$ that results from excluding stochastic CDS liquidity is consistently lower than when CDS liquidity is modeled with a mean of 3.44 bp versus 22.71 bp, a minimum of -88.27 bp (5.08 bp), a maximum of 75.28 bp (73.40 bp), and a standard deviation of 29.95 bp (14.82 bp). The correlation premia $bc$ are similar in both cases: we obtain a mean value of 7.65 bp and a standard deviation of 4.15 bp when CDS liquidity is excluded and 10.45 bp with a standard deviation of 5.79 when CDS liquidity is accounted for. Even if we did not separate the pure liquidity premium and the correlation premium, we would still obtain a negative sum of the two components during the beginning of the observation interval.

We repeat the above analysis for the entire sample, i.e. we re-estimate the default and bond liquidity intensity time-series under the assumption that the CDS market is the liquidity numéraire for each firm. Overall, this re-estimation gives 8,938 negative bond liquidity estimates for 120 firms, 111 of them in the investment grade and all 9 subinvestment grade firms. We then determine the resulting bond credit risk, liquidity, and correlation premia $bd$, $bl$, and $bc$. The resulting mean, standard deviation, minimum, and maximum are displayed in Table V.

A comparison of the estimates in Table V to the original estimates that allow for CDS liquidity in Table III shows that the average bond credit risk premia $bd$ tend to be slightly higher if CDS liquidity is ignored. Across all rating classes, the mean value of $bd$ is higher at 46.26 bp by 1.88 bp than in Table III which is approximately equal to the mean CDS pure liquidity premium $sl$ of 1.94 bp. The main difference, however, lies in the higher averages and the negative minimal values for the bond liquidity premia $bl$. For each rating class, the minimal value of $bl$ is negative with an absolute minimum of -204.39 bp in the BB rating class. A similar result applies for the correlation-induced component of the bond liquidity premium $bc$. In spite of its higher average value of 4.40 bp compared to 3.59 bp when stochastic CDS liquidity is included, the minimal values are consistently negative except for the B and CCC rating class.

These results suggest that neglecting stochastic CDS liquidity yields overestimates of liquidity in the bond market and results in bond price surcharges instead of discounts. At the same time, the default risk is overestimated when the bond liquidity premia become negative, and this results in overestimates of the default probability. Since neglecting CDS liquidity attributes yield differences between the bond and the
CDS market directly to bond liquidity, the effect is especially pronounced when the bond liquidity is high relative to the CDS liquidity. As the CDS market matures, the erroneous results of neglecting CDS liquidity become less striking.

**B. The Impact of Market Factors on Credit Risk, Liquidity, and Correlation Premia**

Our results in Section III.D.2 suggest that the credit risk, liquidity, and correlation premia exhibit a different behavior in the investment and the subinvestment grade sector. In particular, the link between the bond and the CDS market is slightly stronger in the subinvestment grade sector. As a robustness test, we determine in this section whether the time series of the premia components are driven by aggregate market conditions instead of the firm-specific evolution of credit risk and liquidity. To this purpose, we analyze the effect of including market-wide credit risk and liquidity measures in the VECM specification in equation (13) to (15).

As a proxy for credit risk, we choose the S&P Global Corporate Bond Indices for which weekly yield spreads are available on Bloomberg for all rating classes between AAA and B with a constant maturity of 5 years. These indices have two drawbacks for our analysis: they are quoted with regard to bond prices in US-Dollar and are no longer available after May 1, 2007. Nevertheless, the 307 weekly yield observations for each index between June 1, 2001 and May 1, 2007 provide information regarding global credit risk for different rating classes. Liquidity is proxied by the European Central Bank (ECB) financial market liquidity indicator for which daily values were made available to us by the ECB. The indicator is designed to mirror dynamic patterns in the overall liquidity of the financial markets in the Euro area and combines information from the stock, the bond, and the equity options market as well as European interest rate data. A higher value marks higher aggregate financial market liquidity.

Since the time series of the two indices and the estimated premia are not stationary, we estimate a VECM similar to the one in equation (13) to (15) with the credit risk, liquidity, and correlation premia as endogenous variables and the change of the S&P index yield spread and the liquidity indicator as exogenous variables. The results are displayed in Table VI.

As Table VI shows, the inclusion of the aggregate credit risk and liquidity measures hardly affects the dynamics of the firm-specific credit risk premia. In comparison to Table IV, the signs and the significance
level do not change, and even the size of the estimated parameters remains similar in each panel. Both $\Delta bd$ and $\Delta sd$ depend positively on market-wide credit risk and negatively on liquidity. The increase of the adjusted $R^2$ from 15.35\% to 15.36\% for $\Delta bd$ and from 13.22\% to 13.23\% for $\Delta sd$ shows that the explanatory power of the market factors for the entire sample is small. The impact is slightly stronger for the investment grade sector, the coefficient estimates for the credit risk factor double from 0.03 to 0.06 both for $\Delta bd$ and $\Delta sd$, but the explanatory power remains limited. For the subinvestment grade sector, the coefficient estimates are insignificant which implies that credit risk premia in the subinvestment grade sector depend almost completely on the firm’s idiosyncratic default risk.

The inclusion of the market factors into the VECM equations for $bl$ and $sl$ also does not change the signs, the significance level, and the numerical value of the estimated parameters. We obtain a slight positive dependence on credit risk for $\Delta bl$ and a negative one for $\Delta sl$. Furthermore, an improvement of liquidity as measured by the liquidity index has a consistently negative impact on $\Delta bl$ and $\Delta sl$. This result demonstrates that our liquidity definition for the bond and the CDS market reflects an intuitively plausible liquidity. For the investment grade sector, the effect of the market liquidity factor on $\Delta bl$ is more pronounced. We partly attribute this to the fact that the CDS market is, on average, more liquid than the bond market, and partly to the increasing overall liquidity in the CDS market throughout the observation interval. In the subinvestment grade sector, on the other hand, $\Delta bl$ does not react to the market factors at all while $\Delta sl$ exhibits a negative dependence on the liquidity factor which is significant at the 10\% level, suggesting that $sl$ increases when market liquidity decreases. The explanatory power of the market factors is as limited for the liquidity premia as for the credit risk premia.

For the correlation premia, the impact of the aggregate market measures only applies to the bond premia. While $bc$ is significantly affected by the credit risk and liquidity measures, $sc$ is entirely unaffected. This reveals $sc$ to be a pure measure of the firm-specific credit and liquidity risk while $bc$ mirrors $bd$ more closely and thus exhibits a similar dependence on the market factors.

C. The Effect of Increasing and Decreasing Credit Risk

We explore whether the relation between the credit risk, liquidity, and correlation premia across the two markets is sensitive to the state of the bond market as a whole, i.e. whether the premia interact differently in high- and low-risk phases. To do so, we re-estimate the VECM in equation (13) to (15) for increasing
and decreasing credit risk phases separately. Increasing risk phases are defined as time intervals with four consecutive increases in the S&P Global Corporate Bond Index weekly yield spread for that rating class to which a firm belonged during that interval. Decreasing risk phases are analogously defined as intervals with four consecutive decreases. Overall, we obtain 21 four-week intervals with increasing and 17 with decreasing risk for which we analyze the premia on a daily level. The results of the estimation are given in Table VII.

Table VII illustrates that the cointegration relation for the credit risk premia is stable across the increasing and decreasing risk phases. The estimates for $\beta_d$ are close to -1, and the small differences between the increasing and the decreasing risk phase are caused by the impact of the bond liquidity on $sd$. In the increasing credit risk phase and for the subinvestment grade sector, $bl$ tends to be higher, therefore $sd$ is slightly higher, and the estimate of $\beta_d$ is slightly lower. The coefficient estimates for the error correction terms, on the other hand, are considerably higher during phases of increasing credit risk. This result implies that the effect of the bond liquidity on $sd$ becomes less important for deteriorating market conditions which is further supported by the higher adjusted $R^2$ for periods with increasing risk.

Contrary to the credit risk premia, the connection between $\Delta bl$ and $\Delta sl$ differs across periods with increasing and decreasing risk. During increasing risk phases, the cointegration coefficient is positive, hence the liquidity premia tend to move in opposite directions. For the subinvestment grade sector this also applies when credit risk decreases. Investment grade liquidity premia, on the other hand, move in the same direction when credit risk decreases, which agrees with our graphical results in Figure 2. On comparing the error correction coefficients, we observe that $\Delta bl$ is affected more strongly in decreasing and $\Delta sl$ in increasing risk phases. For $\Delta sl$, this is true both for the investment grade and the subinvestment grade sector, but $\Delta bl$ is not significantly affected by the error correction term in the investment grade sector. This is further evidence that the liquidity of the investment grade bond market is unaffected by that of the CDS market while the reverse is not true. For the subinvestment grade sector, liquidity premium deviations in one market have a consistently reverse effect on the other market’s liquidity. As shown before, the bond market reacts less strongly than the CDS market. In particular when risk decreases, the sensitivity of the bond market decreases and that of the CDS market increases on an absolute level.
The negative estimate for the cointegration coefficient of $bc$ and $sc$ is consistent with our earlier finding that correlation premia are mostly due to the effect of the credit risk factor on the liquidity intensity. Interestingly, the absolute value of the cointegration coefficient is higher when credit risk decreases. We take this as a sign that the dynamics of the CDS bid and ask premia become more dissimilar in increasing risk phases, therefore a smaller fraction of the CDS premia can be attributed to the impact of the default intensity on the liquidity intensity. Our above results suggest that protection sellers may increase ask premia disproportionately when credit risk increases which we cannot fully capture by means of our time-invariant correlation coefficient.

D. Estimation from Ask or Bid CDS Premia

In Section III, we use the CDS ask and bid premia simultaneously to extract the pure credit risk, the liquidity, and the correlation component from CDS premia and bond yield spreads. However, only the sum of these components can be observed in practice and our estimate may differ significantly from the true values. As a robustness test, we repeat the firm-specific estimation procedure described in Section III.B once using only CDS ask premia and once using only CDS bid premia instead of both. We then compare the resulting estimates of $bd$, $bl$, $bc$, $sd$, $sl$, and $sc$ with those we obtain for the entire sample. In particular, we compute the mean, the standard deviation, and the mean absolute difference between the estimates which are obtained using only one CDS premium and the estimates which use both simultaneously. The results are displayed in Table VIII.

Table VIII shows that the estimates of the credit risk, liquidity, and correlation premia are almost identical regardless of which CDS premia are used in the estimation. On average, the mean estimate of $bd$ from the CDS ask premium of 44.15 bp falls below the one using both premia by 0.23 bp, but the similar standard deviation and the mean absolute deviation of 0.48 bp imply that the sign is not indicative of a systematic error. The same is true for the estimate which uses only the bid premia with a mean difference between the credit risk premia of 0.40 bp and a mean absolute error of 0.55 bp. For the bond liquidity premia $bl$, we obtain the reverse result, the mean estimates which only use ask premia are slightly higher and the ones using only bid premia are slightly lower. The difference, however, does not appear to be systematic which
is supported by the absolute mean deviation of 1.37 bp and 1.42 bp. A similar pattern as for \( bd \) applies for \( bc \). The results for the CDS pure credit risk premia \( sd \) are also similar to those for the bond. Again, the use of ask premia leads to a very slight underestimation of the credit risk premia while bid premia yield slightly higher values. The largest differences are caused for the CDS pure liquidity and correlation premia \( sl \) and \( sc \). This agrees with the earlier result that the correlation of the CDS ask and bid liquidity intensities with the default intensity differs. For ask premia, we find lower pure liquidity premia and higher correlation premia. Bid premia for which the liquidity intensity is less strongly correlated with the default intensity yield higher liquidity premia and lower correlation premia.

Overall, we find that the choice of ask or bid premia in the CDS market does not significantly affect the size and the dynamics of the estimated premia. Since these premia are not directly observable in the market, we take the robust behavior of the estimates as a sign that our estimation does not result in a systematic deviation from the true premia.

V. Summary and Conclusion

The purpose of our paper was to develop a credit risk model that simultaneously accounts for stochastic liquidity in bond and in CDS markets. While there is broad agreement in the literature that modeling liquidity in bond prices is an important issue both in structural and in reduced-form models, CDS markets are often assumed to be perfectly liquid. Therefore, default probabilities or, in the context of reduced-form models, default intensities, are estimated directly from observed CDS mid premia and the results are used to measure the size of the default component in corporate bond prices and yield spreads.

In our paper, we develop a model that explicitly allows for stochastic liquidity in CDS ask and bid premia. As CDS are derivatives and not assets, it is not clear whether illiquidity should consistently increase mid premia or whether it ought only to result in larger bid-ask-spreads. We avoid this issue by directly modeling the CDS ask and bid premium. This approach allows for closed-form bond prices and CDS premia which are affected by the identical default risk but are subject to different liquidity risk factors. Specifically, we determine a theoretical, liquidity-free yield spread for par bonds and CDS premia unaffected by the CDS-specific liquidity. These credit risk premia can be compared to the corresponding liquidity and correlation premia and the CDS mid premium as well as to the yield spread. The latter two can be – and our empirical analysis shows them to be – affected by illiquidity and the correlation between credit risk and liquidity.
We estimate the model using bond mid prices and CDS bid and ask premia for firms with ratings between AAA and CCC from a broad range of sectors. As the default-free liquidity numéraire, we use the market for German Government Bonds. Our results show that the bond and CDS markets as a whole reacted strongly to the Enron and WorldCom defaults. The estimated credit risk component in CDS premia and bond credit spreads is almost identical. In the bond market, it amounts to 60% of the observed credit spreads while 95% of the CDS mid premia are due to pure credit risk. The pure liquidity premia constitute 35% of the credit spread and 4% of the CDS premia, and the correlation component amounts to 5% and 1%, respectively. We also find that the period of the highest credit risk coincides with a period of low liquidity in the bond market, both with regard to the pure liquidity and to the correlation premium. Liquidity in the CDS market exhibits a less straightforward behavior, but the estimate of the pure credit risk component in the CDS market becomes more biased towards the bid in times of high credit risk for investment-grade CDS. Economically, this suggests that protection sellers demand an additional premium in excess of the “fair” credit risk premium when setting their ask quotes. In the subinvestment grade sample, the evidence is mixed. On the one hand, the subinvestment grade CDS market shows a higher average liquidity and correlation premium than the investment grade CDS market. On the other hand, the pure liquidity premia become negative as credit risk rises for badly rated debt. This implies that investors increasingly use the CDS market to take on synthetic credit risk as the liquidity of the bond market dries up.

Throughout our observation interval from June 1, 2001 to June 30, 2007, we observe declining default risk, increasing bond market liquidity, and increasing liquidity jointly with a more symmetrical distribution of liquidity premia in the CDS market. From an economic perspective, this agrees with the evolution and the standardization of the CDS market. The asymmetrical distribution of the liquidity premia between protection buyers and sellers in the CDS market indicates that CDS mid premia are not an appropriate measure of the pure credit risk component. In a robustness analysis, we demonstrate that restricting stochastic liquidity to the bond market can result in price surcharges and yield discounts for corporate bonds. This effect, however, becomes less apparent as CDS market liquidity evolves.

We find evidence of an empirical relation between the liquidity of the bond and the CDS market in excess of the liquidity spill-over which is immanent to our model. Specifically, we observe that a change in bond liquidity affects CDS liquidity premia in the investment grade sector in the opposite direction but that the reverse effect does not apply. In the subinvestment grade sector, the effect is bilateral but remains more pronounced for CDS, suggesting that the CDS market becomes a more attractive substitute for taking on
credit risk synthetically when the bond market is illiquid. Our results also imply that the investment grade bond and CDS sectors are less integrated than the subinvestment grade sectors. Overall market conditions, on the other hand, affect the investment grade sector more strongly. Lastly, our analysis reveals that the relation between the liquidity premia across the two markets differs during periods of increasing and decreasing default risk.

An issue not addressed in this paper is that CDS contracts are usually designed to allow for a number of bonds deliverable upon the default of a given reference asset. Before default, however, it is not clear which bond will be cheapest to deliver. Jankowitsch, Pullirsch, and Veza (2007) show that this choice option of the protection buyer is also priced in CDS premia. As a second extension of our model, it is also possible to add information from the stock market. Blanco, Brennan, and Marsh (2005) and Norden and Weber (2004) have explored the information spillover between stock, CDS and bond markets, and their results suggest that incorporating stock market information may facilitate the estimation of the default intensity. It may be interesting to explore whether including this information in a reduced-form model will render explicit liquidity-modeling in CDS premia unnecessary or whether, as our results suggest, our proposed extension of the existing reduced-form models is vital if information from the CDS market is used.
Appendix A. Analytical Solutions for the Discount Factors

The dynamics of the default and liquidity intensities are defined as follows. First, we define the latent risk factors $x$ and $y^l$ through the following system of stochastic differential equations:

\[
\begin{pmatrix}
    dx(t) \\
    dy^b(t) \\
    dy^{ask}(t) \\
    dy^{bid}(t)
\end{pmatrix} = \begin{pmatrix}
    \alpha - \beta x(t) \\
    \mu^b \\
    \mu^{ask} \\
    \mu^{bid}
\end{pmatrix} dt + \begin{pmatrix}
    \sigma \sqrt{x(t)} dW_x(t) \\
    \eta^b dW^b_{\tau} \\
    \eta^{ask} dW^{ask}_{\tau} \\
    \eta^{bid} dW^{bid}_{\tau}
\end{pmatrix},
\]  

(A.1)

with constants $\alpha, \beta, \sigma, \mu^l, \eta_l$, and Brownian motions $W_x$ and $W^l_{\tau}, l \in \{b, as, bid\}$. The Brownian motions governing $x$ and $y^l, l \in \{b, as, bid\}$ are independent. The intensities $\lambda$ and $\gamma$ are then defined as linear combinations of the latent factors

\[
\begin{pmatrix}
    \lambda(t) \\
    \gamma^b(t) \\
    \gamma^{ask}(t) \\
    \gamma^{bid}(t)
\end{pmatrix} = \begin{pmatrix}
    x(t) + g_b \cdot y^b(t) + g_{ask} y^{ask}(t) + g_{bid} y^{bid}(t) \\
    f_b \cdot x(t) + y^b(t) + \omega_b,ask y^{ask}(t) + \omega_b,bid y^{bid}(t) \\
    f_{ask} \cdot dx(t) + \omega_b,ask y^{ask}(t) + \omega_{ask},bid y^{bid}(t) \\
    f_{bid} \cdot dx(t) + \omega_b,bid y^{bid}(t) + \omega_{ask,bid} y^{ask}(t) + \gamma^{bid}(t)
\end{pmatrix}.
\]  

(A.2)

We only show the derivation for $E_t [\tilde{P}(t, \tau) \tilde{L}^b(t, \tau)]$, the other pricing factors are derived in the same way.

The joint expectation for the discount factors $\tilde{P}(t, \tau)$ and $\tilde{L}^b(t, \tau)$ is given by:

\[
E_t [\tilde{P}(t, \tau) \cdot \tilde{L}^b(t, \tau)] = E_t \left[ \exp \left( - \int_t^\tau \lambda(s) + \gamma^b(s) ds \right) \right]
\]

\[
= E_t \left[ \exp \left( - \int_t^\tau (1 + f_b) x(s) + (1 + g_b) y^b(s) + (g_{ask} + \omega_{b,ask}) y^{ask}(s) + (g_{bid} + \omega_{b,bid}) y^{bid}(s) ds \right) \right]
\]

\[
= E_t \left[ \exp \left( - \int_t^\tau (1 + f_b) x(s) ds \right) \right] \cdot E_t \left[ \exp \left( - \int_t^\tau (1 + g_b) y^b(s) ds \right) \right]
\]

\[
\cdot E_t \left[ \exp \left( - \int_t^\tau (g_{ask} + \omega_{b,ask}) y^{ask}(s) ds \right) \right] \cdot E_t \left[ \exp \left( - \int_t^\tau (g_{bid} + \omega_{b,bid}) y^{bid}(s) ds \right) \right]
\]

\[
= : \tilde{P}(t, \tau, x; 1 + f_b) \tilde{L}(t, \tau, y^b; 1 + g_b) \tilde{L}(t, \tau, y^{ask}; g_{ask} + \omega_{b,ask}) \tilde{L}(t, \tau, y^{bid}; g_{bid} + \omega_{b,bid})
\]

(A.3)
where $P^b$ and $L^b$ are the solutions referenced in section I.C. The dynamics of the scaled latent risk factors $(1+f_b)x$, $(1+g_b)y^b$, $(g_{ask} + \omega_{b,ask})y^{ask}$, and $(g_{bid} + \omega_{b,bid})y^{bid}$ are identical to those of the original latent factors with the process parameters adjusted:

\[
\begin{pmatrix}
    d(1+f_b)x(t) \\
    d(1+g_b)y^b(t) \\
    d(g_{ask} + \omega_{b,ask})y^{ask}(t) \\
    d(g_{bid} + \omega_{b,bid})y^{bid}(t)
\end{pmatrix}
= \begin{pmatrix}
    (1+f_b)\alpha - \beta(1+f_b)x(t) \\
    (1+g_b)\mu^b \\
    (g_{ask} + \omega_{b,ask})\mu^{ask} \\
    (g_{bid} + \omega_{b,bid})\mu^{bid}
\end{pmatrix} dt + \begin{pmatrix}
    \sqrt{1+f_b}\sigma \sqrt{(1+f_b)x(t)} dW_x(t) \\
    (1+g_b)\eta^b dW_y(t) \\
    (g_{ask} + \omega_{b,ask})\eta^{ask} dW_{y^{ask}}(t) \\
    (g_{bid} + \omega_{b,bid})\eta^{bid} dW_{y^{bid}}(t)
\end{pmatrix}
\]

with initial values $(1+f_b)x_0$, $(1+g_b)y^b_0$, $(g_{bid} + \omega_{b,bid})y^{bid}_0$, and $(g_{bid} + \omega_{b,bid})y^{bid}_0$.

Thus, the following well-known analytical solutions arise:

\[
P(t, \tau; x; k) := a_1(t, \tau; k) \cdot \exp \left[ -a_2(t, \tau; k) \cdot kx(t) \right],
\]

\[
L(t, \tau; y; k) := a_3(t, \tau; k) \cdot \exp \left[ -a_4(t, \tau; k) \cdot ky(t) \right],
\]

where

\[
a_1(t, \tau; k) = \left( \frac{1 - \kappa(k)}{1 - \kappa(k)\exp[\phi(k)(\tau-t)]} \right)^{\frac{2\kappa}{\sigma^2}} \exp \left[ \frac{\alpha(\beta + \phi(k))}{\sigma^2} (\tau-t) \right],
\]

\[
a_2(t, \tau; k) = \frac{\phi(k) - \beta}{\sigma^2 k} + \frac{2\phi(k)}{\sigma^2 k(\kappa(k)\exp[\phi(k)(\tau-t)] - 1)},
\]

\[
a_3(t, \tau; k) = \exp \left[ \frac{k^2\eta^2}{6}(\tau-t)^3 + \frac{k\mu}{6}(\tau-t)^2 \right],
\]

\[
a_4(t, \tau; k) = \tau - t,
\]

\[
\phi(k) = \sqrt{2\sigma^2 k + \beta^2}, \ \kappa(k) = \frac{\beta + \phi(k)}{\beta - \phi(k)}.
\]
The bond and CDS pricing equations also contain $E_t[\tilde{P}(t, \tau_{t-1}) \cdot \tilde{L}^b(t, \tau_t)]$. Since $\lambda$ and $\gamma$ are correlated, we also have to determine the expectation of this non-simultaneous discount factor. Wlg, assume that $\tau_{t-1} = \tau_1$ and $\tau_t = \tau_2$, $\tau_1 \leq \tau_2$. Then, the definition of $\tilde{P}$ and $\tilde{L}^b$ implies that

$$
\tilde{P}(t, \tau_1) \cdot \tilde{L}^b(t, \tau_2) = \exp \left( -\int_t^{\tau_1} x(s) + g_b y^b(s) + g_{ask} y^{ask}(s) + g_{bid} y^{bid}(s) \ ds \right.
- \int_t^{\tau_2} f_b x(s) + y^b(s) + \omega_{b,ask} y^{ask}(s) + \omega_{b,bid} y^{bid}(s) \ ds
\right) = \exp \left( -\int_t^{\tau_1} (1 + f_b) x(s) \ ds - \int_{\tau_1}^{\tau_2} f_b x(s) \ ds \right.
- \int_t^{\tau_1} (1 + g_b) y^b(s) \ ds - \int_{\tau_1}^{\tau_2} y^b(s) \ ds
- \int_t^{\tau_1} (g_{ask} + \omega_{b,ask}) y^{ask}(s) \ ds - \int_{\tau_1}^{\tau_2} \omega_{b,ask} y^{ask}(s) \ ds
- \int_t^{\tau_1} (g_{bid} + \omega_{b,bid}) y^{bid}(s) \ ds - \int_{\tau_1}^{\tau_2} \omega_{b,bid} y^{bid}(s) \ ds \right). \tag{A.7}
$$

The independence of the latent factors allows us to split up the joint expectation as

$$
E_t \left[ \tilde{P}(t, \tau_1) \cdot \tilde{L}^b(t, \tau_2) \right] = E_t \left[ \exp \left( -\int_t^{\tau_1} (1 + f_b) x(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} f_b x(s) \ ds \right) \right]
:= P(t, \tau_1, \tau_2; \tau_{t-1}, f_b)
\cdot E_t \left[ \exp \left( -\int_t^{\tau_1} (1 + g_b) y^b(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} y^b(s) \ ds \right) \right]
\cdot E_t \left[ \exp \left( -\int_t^{\tau_1} (g_{ask} + \omega_{b,ask}) y^{ask}(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} \omega_{b,ask} y^{ask}(s) \ ds \right) \right]
\cdot E_t \left[ \exp \left( -\int_t^{\tau_1} (g_{bid} + \omega_{b,bid}) y^{bid}(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} \omega_{b,bid} y^{bid}(s) \ ds \right) \right]. \tag{A.8}
$$

Applying the law of iterated expectation, we obtain

$$
E_t \left[ \exp \left( -\int_t^{\tau_1} (1 + f_b) x(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} f_b x(s) \ ds \right) \right] = E_t \left[ \exp \left( -\int_t^{\tau_1} (1 + f_b) x(s) \ ds \right) \exp \left( -\int_{\tau_1}^{\tau_2} f_b x(s) \ ds \right) \right] = a_1(\tau_1, \tau_2; f_b) E_t \left[ \exp \left( -a_2(\tau_1, \tau_2; f_b) f_b x(\tau_1) \right) \exp \left( -\int_t^{\tau_1} (1 + f_b) x(s) \ ds \right) \right], \tag{A.9}
$$

35
which, by the moment-generating function of $x$, has the following exponential-affine solution

$$P(t, \tau_1, \tau_2, x; f_b, 1 + f_b) = a_1(\tau_1, \tau_2; f_b) b_1(t, \tau_1, \tau_2; f_b, 1 + f_b) \exp \left[-b_2(t, \tau_1, \tau_2; f_b, 1 + f_b) x(t) \right], \quad (A.10)$$

where $\phi, a_1,$ and $a_2$ are defined as above and

$$b_1(t, \tau_1, \tau_2; k_1, k_2) = \frac{2\phi(k_2) \exp \left[ \frac{\tau_1 - \tau_2}{2} (\phi(k_2) + \beta) \right]}{\sigma^2 a_2(\tau_1, \tau_2; k_1) k_1 (\exp [\phi(k_2)(\tau_1 - t)] - 1) + \phi(k_2) - 2 + \exp [\phi(k_2)(\tau_1 - t)] (\phi(k_2) + \beta)},$$

$$b_2(t, \tau_1, \tau_2; k_1, k_2) = \frac{a_2(\tau_1, \tau_2; k_1) k_1 [\phi(k_2) + 2 + \exp [\phi(k_2)(\tau_1 - t)] (\phi(k_2) - \beta)] + 2k_2 (\exp [\phi(k_2)(\tau_1 - t)] - 1)}{\sigma^2 a_2(\tau_1, \tau_2; k_1) k_1 (\exp [\phi(k_2)(\tau_1 - t)] - 1) + \phi(k_2) - 2 + \exp [\phi(k_2)(\tau_1 - t)] (\phi(k_2) + \beta)}.$$

Simultaneously, we obtain

$$E_t \left[ \exp \left(-\int_t^{\tau_1} k_1 y^i(s) \, ds \right) \exp \left(-\int_{\tau_1}^{\tau_2} k_2 y^j(s) \, ds \right) \right] = a_3(\tau_1, \tau_2; k_2) E_t \left[ \exp \left(-a_4(\tau_1, \tau_2; k_2) k_2 y^j(\tau_1) \right) \exp \left(-\int_t^{\tau_1} k_1 y^i(s) \, ds \right) \right], \quad (A.11)$$

which has the exponential-affine solution

$$L^i(t, \tau_1, \tau_2; y^i; k_1, k_2) = a_3(\tau_1, \tau_2; k_2) b^i_3(t, \tau_1, \tau_2; k_1, k_2) \exp \left[-b^i_4(t, \tau_1, \tau_2; k_1, k_2) y^i(t) \right], \quad (A.12)$$

where $a_3^i$ and $a_4^i$ are defined as above and

$$b^i_3(t, \tau_1, \tau_2; k_1, k_2) = \exp \left[ \eta^2 k_1^2 (\tau_1 - t)^3 + \eta^2 k_2 a_4(\tau_1, \tau_2; k_2) - \mu^i k_1 (\tau_1 - t)^2 \right.$$

$$\left. + \left( \eta^2 k_2 a_4(\tau_1, \tau_2; k_2) - \mu^i \right) a_4(\tau_1, \tau_2; k_2) k_2 (\tau_1 - t) \right],$$

$$b^i_4(t, \tau_1, \tau_2; k_1, k_2) = a_4(\tau_1, \tau_2; k_2) k_2 + k_1 (\tau_1 - t).$$
References


Huang, Jing-Zhi, and Ming Huang, 2003, How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?, Working Paper, Graduate School of Business, Stanford University.


Notes

1 As an alternative, we also used the swap curve which is, on average, 10 basis points higher than the Nelson-Siegel-Svensson curve for a maturity of 5 years. Since the dynamics are almost identical, we only present the results for the German Government Bonds.

2 It is natural that bonds of the same issuer with identical seniority have the same default probability during the next infinitesimally small time interval. The liquidity, on the other hand, may well depend on the maturity and the coupon of a bond. We expect the homogeneity of the bonds of the same issuer to limit the differences. In addition, the functional form of the stochastic liquidity process results in larger liquidity premia for bonds with a longer maturity.

3 Convergence is usually achieved in less than 10 iteration steps.

4 The Nielsen Company is active in marketing and media information, business publications, and trade shows in over 100 countries and has a total of 42,000 employees.

5 For a detailed description of the indicator, see European Central Bank (2007).

6 Alternatively, we have used the JP Morgan Aggregate Index Europe asset swap rates and the ICMA European Corporate Bond All Maturities Yield Index for which Bloomberg provides daily values to define the increasing and the decreasing risk phases. The results were virtually identical.
Figure 1. Average Bond Yield Spreads and Mid CDS Premia Time Series

The figure depicts the average bond yield spreads and mid CDS premia between June 1, 2001 and June 30, 2007. Bond spreads are computed as a bond’s yield-to-maturity computed from the mid price less the theoretical yield-to-maturity of a bond with identical coupon and maturity computed from the Nelson-Siegel-Svensson curve for German Government Bonds. The bond spreads are subsequently interpolated to obtain a synthetic 5-year maturity. Averages are taken across all firms which were rated investment grade or subinvestment grade, respectively, on a given date. Average yield spreads are denoted in black, mid CDS premia in grey. The solid line is used to depict the investment grade, the dashed line to depict the subinvestment grade time series.
**Figure 2. Estimated Credit Risk, Liquidity, and Correlation Premia Time Series**

The figure depicts the model-implied premia components for the investment and subinvestment grade sector between June 1, 2001 and June 30, 2007. The bond premia are yield spreads for a synthetical 5-year par bond, the CDS premia are mid premia for a 5-year contract. The credit risk premia (solid black line) reflect the impact of the credit risk factor (and the bond liquidity for the CDS). The liquidity premium (dotted grey line) reflect the impact of the instrument-specific liquidity factor for without the impact of the latent credit risk factor on the liquidity intensity. The correlation premia (dashed grey line) reflect the impact of the credit risk factor on the liquidity intensities. Averages are computed across the investment and subinvestment grade sector on each date. All values are in basis points.

**Panel A: Bond Premia**

- **Investment Grade Premia Bond**
- **Subinvestment Grade Premia Bond**
Panel B: CDS Premia

Investment Grade Premia CDS

Subinvestment Grade Premia CDS
Figure 3. The Effect of Excluding Stochastic CDS Liquidity

The figure depicts the model-implied credit risk, liquidity, and correlation components of the bond premia for the communications company Nielsen between June 1, 2001 and June 30, 2007. The bond premia are yield spreads for a synthetical 5-year par bond. The model is estimated once ignoring stochastic liquidity in CDS premia (solid lines) and once including stochastic liquidity in CDS premia (dashed and dotted lines). The credit risk premia are depicted in black (dashed when CDS liquidity is included in the estimation), the liquidity premia in dark grey (dotted when CDS liquidity is included in the estimation), the correlation premia in light grey (dashed when CDS liquidity is included in the estimation). All values are in basis points.
Table 1: **Firms by Rating Class and Industry Sector**

The table presents the number of firms in each rating class and industry group. Ratings are averages for the firm over all dates on which both CDS ask and bid premia and at least 2 bond prices were observed. The last columns and rows show the number of mid bond prices and mid CDS premia for each industry group and rating class in our sample between June 1, 2001 and June 30, 2007.

<table>
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<tr>
<th>Industry Sector</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>All</th>
<th># Obs. Bonds</th>
<th># Obs. CDS</th>
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<td>13,079</td>
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<td>3</td>
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<td>2</td>
<td>-</td>
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<td>47,497</td>
<td>15,634</td>
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<tr>
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<td>2</td>
<td>-</td>
<td>-</td>
<td>4</td>
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<td>-</td>
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<td>-</td>
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<td>19,036</td>
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<tr>
<td>All</td>
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<td>32</td>
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<td>47</td>
<td>8</td>
<td>1</td>
<td>155</td>
<td>497,254</td>
<td>131,222</td>
</tr>
</tbody>
</table>

| # Obs. Bonds                   | 3,552| 106,206| 116,359| 248,343| 21,238| 1,556| 497,254 |
| # Obs. CDS                     | 1,085| 27,015 | 53,203 | 41,338 | 7,842 | 739  | 131,222 |
Table II  
Factor Sensitivities

The table presents the estimates for the factor sensitivities. $g_b$, $g_{ask}$, and $g_{bid}$ measure the impact of the latent bond, CDS ask, and CDS bid liquidity factors $y^b$, $y^{ask}$, and $y^{bid}$ on the default intensity $\lambda$. $f_b$, $f_{ask}$, and $f_{bid}$ measure the impact of the latent credit risk factor $x$ on the bond, CDS ask, and CDS bid liquidity intensities $\gamma^b$, $\gamma^{ask}$, and $\gamma^{bid}$. $\omega_{b,ask}$, $\omega_{b,bid}$, and $\omega_{ask,bid}$ measure the cross-impact of the latent bond, CDS ask, and CDS bid liquidity factors on the bond, CDS ask, and CDS bid liquidity intensities $\gamma^b$, $\gamma^{ask}$, and $\gamma^{bid}$. The first row of each panel gives the number of firms for which the sensitivity estimate was significantly different from 0, the second row the number of estimates significantly larger than 0, the third row the number of estimates significantly smaller than 0. The fourth and fifth row present the mean estimate and the standard deviation. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms.

<table>
<thead>
<tr>
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<th>Panel A: All</th>
<th>Panel B: Investment Grade</th>
<th>Panel B: Subinvestment Grade</th>
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<td>137</td>
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<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>Mean</td>
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<td>0.37***</td>
<td>-0.07***</td>
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<td>0.03</td>
<td>0.04</td>
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45
Table III  
Estimated Credit Risk, Liquidity, and Correlation Premia

The table presents the mean, standard deviation, minimum, and maximum of the model-implied premia components for each rating class. $bd$ gives the pure credit risk, $bl$ the pure liquidity, and $bc$ the correlation component in the yield spread of a synthetical 5-year par bond. $sd$ gives the pure credit risk, $sl$ the pure liquidity, and $sc$ the correlation component in the mid premium for a 5-year CDS contract. The standard deviation, minimum, and maximum are determined both over time and across observations within the rating class on each date. All values are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
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<td>2.96</td>
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<td>126.95</td>
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<td>24.92</td>
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<td>50.65</td>
<td>61.85</td>
<td>3.09</td>
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<td>30.12</td>
<td>43.95</td>
<td>55.77</td>
<td>54.77</td>
<td>58.19</td>
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<td>16.89</td>
<td>19.51</td>
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<td>3.59</td>
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<td>1.48</td>
<td>1.50</td>
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<td>349.97</td>
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<td>9.00</td>
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<td>-152.59</td>
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<td>-153.82</td>
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<td>8.97</td>
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<td>0.19</td>
<td>0.45</td>
<td>5.41</td>
<td>8.48</td>
<td>5.57</td>
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<tr>
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<td>max($sc$)</td>
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<td>98.85</td>
<td>43.77</td>
<td>18.17</td>
<td>98.85</td>
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</table>
The Dynamic Relationship of Credit Risk, Liquidity, and Correlation Premia

Table IV

The table presents the estimated coefficients for the vector error correction model (VECM) in equation (13), (14), and (15). $bd$ is the pure credit risk, $bl$ the pure liquidity, and $bc$ the correlation component in the yield spread of a synthetical 5-year par bond. $sd$ gives the pure credit risk, $sl$ the pure liquidity, and $sc$ the correlation component in the mid premium for a 5-year CDS contract. The dependent variables are the premium changes, the explanatory variables are the vector error correction terms $(bd_{t-1} + \beta_{bd}sd_{t-1})$, $(bl_{t-1} + \beta_{bl}sl_{t-1})$, and $(bc_{t-1} + \beta_{bc}sc_{t-1})$, and the lagged premia changes. $\beta$ denotes the cointegration coefficient, $\alpha = (\alpha_{bd}, \alpha_{sl})$ the coefficient vector of the error correction term, and $\Gamma_{bd}$ the coefficient vector of the lagged bond (b) and CDS (s) premium changes with $\Gamma_{bd} = (\Gamma_{bd,b}, \Gamma_{bd,s})$, $\Gamma_{sl} = (\Gamma_{sl,b}, \Gamma_{sl,s})$. The top row of each panel displays the number of firms for which a) the augmented Dickey-Fuller test could not reject a unit root in the premia time series at the 10% significance level, b) the augmented Dickey-Fuller test could reject a unit root in the first differences at the 10% level, c) the Johansen test could not reject cointegration of the time series at the 10% level, d) the augmented Dickey-Fuller could reject a unit root in the residuals of the VECM at the 10% level. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms. Coefficients are given for premia in basis points, the adjusted $R^2$ is in percentage points.

Panel A: All

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<th></th>
<th>$\Delta bd$</th>
<th>$\Delta sl$</th>
<th>$\Delta bl$</th>
<th>$\Delta sl$</th>
<th>$\Delta bc$</th>
<th>$\Delta sc$</th>
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<td>143</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.00***</td>
<td>9.38***</td>
<td>-62.92***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>-0.05*</td>
<td>-0.15***</td>
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<td>$\Gamma_{bd}$</td>
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<td>0.31***</td>
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<td>Adj. $R^2$</td>
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<td>-0.01***</td>
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<tr>
<td>$\Gamma_{bd}$</td>
<td>-0.40***</td>
<td>0.08***</td>
<td>-0.33***</td>
<td>-0.01***</td>
<td>-0.15***</td>
<td>0.01**</td>
</tr>
<tr>
<td>$\Gamma_{sl}$</td>
<td>0.32***</td>
<td>-0.41***</td>
<td>-0.06</td>
<td>-0.40***</td>
<td>0.24***</td>
<td>-0.03***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>8.75</td>
<td>7.68</td>
<td>27.25</td>
<td>37.88</td>
<td>12.28</td>
<td>2.45</td>
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</table>

Panel C: Subinvestment Grade

<table>
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<th>$\Delta bd$</th>
<th>$\Delta sl$</th>
<th>$\Delta bl$</th>
<th>$\Delta sl$</th>
<th>$\Delta bc$</th>
<th>$\Delta sc$</th>
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<td>5</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>-0.98***</td>
<td>17.29***</td>
<td>-5.96***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.35***</td>
<td>-1.03***</td>
<td>-0.16***</td>
<td>-0.26***</td>
<td>-0.10***</td>
<td>0.02***</td>
</tr>
<tr>
<td>$\Gamma_{bd}$</td>
<td>-0.87***</td>
<td>0.76***</td>
<td>-0.26***</td>
<td>-0.02***</td>
<td>0.01***</td>
<td>0.04**</td>
</tr>
<tr>
<td>$\Gamma_{sl}$</td>
<td>0.23***</td>
<td>-0.92***</td>
<td>-0.01</td>
<td>-0.16***</td>
<td>0.02***</td>
<td>-0.07***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>21.99</td>
<td>19.90</td>
<td>21.74</td>
<td>33.59</td>
<td>5.70</td>
<td>5.54</td>
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</table>
Table V
Credit Risk, Liquidity, and Correlation Premia Without CDS Liquidity

The table presents the mean, standard deviation, minimum, and maximum of the model-implied premia components for each rating class when stochastic liquidity in the CDS market is ignored. $bd$ gives the pure credit risk, $bl$ the pure liquidity, and $bc$ the correlation component in the yield spread of a synthetical 5-year par bond. $sd$ gives the mid premium for a 5-year CDS contract which is assumed to reflect only credit risk. The standard deviation, minimum, and maximum are determined both over time and across observations within the rating class on each date. All values are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bd$</td>
<td>6.18</td>
<td>13.72</td>
<td>28.05</td>
<td>54.89</td>
<td>249.92</td>
<td>348.59</td>
<td>272.20</td>
<td>46.26</td>
</tr>
<tr>
<td>Std. Dev.($bd$)</td>
<td>4.41</td>
<td>18.30</td>
<td>28.36</td>
<td>64.67</td>
<td>238.09</td>
<td>163.18</td>
<td>33.73</td>
<td>82.61</td>
</tr>
<tr>
<td>min($bd$)</td>
<td>4.04</td>
<td>3.38</td>
<td>3.95</td>
<td>4.04</td>
<td>33.60</td>
<td>33.98</td>
<td>209.11</td>
<td>3.38</td>
</tr>
<tr>
<td>max($bd$)</td>
<td>51.93</td>
<td>255.31</td>
<td>351.75</td>
<td>1,214.12</td>
<td>1,807.87</td>
<td>1,126.66</td>
<td>386.34</td>
<td>1,806.87</td>
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<tr>
<td>$bl$</td>
<td>0.11</td>
<td>11.82</td>
<td>25.63</td>
<td>31.11</td>
<td>49.12</td>
<td>56.63</td>
<td>1.58</td>
<td>24.65</td>
</tr>
<tr>
<td>Std. Dev.($bl$)</td>
<td>1.70</td>
<td>36.47</td>
<td>52.32</td>
<td>58.23</td>
<td>60.46</td>
<td>63.38</td>
<td>12.05</td>
<td>50.74</td>
</tr>
<tr>
<td>min($bl$)</td>
<td>-2.28</td>
<td>-120.48</td>
<td>-18.62</td>
<td>-200.82</td>
<td>-204.39</td>
<td>-150.95</td>
<td>-16.68</td>
<td>-204.39</td>
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<td>max($bl$)</td>
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<td>109.36</td>
<td>485.57</td>
<td>328.30</td>
<td>452.84</td>
<td>281.89</td>
<td>105.82</td>
<td>485.57</td>
</tr>
<tr>
<td>$bc$</td>
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<td>-0.08</td>
<td>2.62</td>
<td>10.05</td>
<td>15.04</td>
<td>21.20</td>
<td>1.80</td>
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<td>Std. Dev.($bc$)</td>
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<td>6.55</td>
<td>15.23</td>
<td>24.31</td>
<td>14.16</td>
<td>0.87</td>
<td>10.05</td>
</tr>
<tr>
<td>min($bc$)</td>
<td>-1.35</td>
<td>-1.25</td>
<td>-1.00</td>
<td>-1.10</td>
<td>-1.83</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.83</td>
</tr>
<tr>
<td>max($bc$)</td>
<td>0.18</td>
<td>25.76</td>
<td>193.46</td>
<td>522.07</td>
<td>325.42</td>
<td>112.67</td>
<td>8.62</td>
<td>522.07</td>
</tr>
<tr>
<td>$sd$</td>
<td>6.20</td>
<td>15.57</td>
<td>31.04</td>
<td>58.99</td>
<td>259.22</td>
<td>367.48</td>
<td>273.55</td>
<td>47.12</td>
</tr>
<tr>
<td>Std. Dev.($sd$)</td>
<td>4.57</td>
<td>11.13</td>
<td>29.59</td>
<td>68.36</td>
<td>237.40</td>
<td>164.85</td>
<td>30.06</td>
<td>85.87</td>
</tr>
<tr>
<td>min($sd$)</td>
<td>4.71</td>
<td>4.18</td>
<td>4.77</td>
<td>4.80</td>
<td>36.71</td>
<td>37.11</td>
<td>211.53</td>
<td>4.18</td>
</tr>
<tr>
<td>max($sd$)</td>
<td>53.58</td>
<td>256.11</td>
<td>355.75</td>
<td>1,302.80</td>
<td>1,849.63</td>
<td>1,158.31</td>
<td>400.66</td>
<td>1,849.63</td>
</tr>
</tbody>
</table>
### Table VI

**Impact of Market-Wide Credit Risk and Liquidity Factors**

The table presents the estimated coefficients for the VECM with exogenous variables. $bd$ is the pure credit risk, $bl$ the pure liquidity, and $bc$ the correlation component in the yield spread of a synthetical 5-year par bond. $sd$ gives the pure credit risk, $sl$ the pure liquidity, and $sc$ the correlation component in the mid premium for a 5-year CDS contract. The dependent variables are the premium changes, the explanatory variables are the vector error correction terms, the lagged premia changes, and the exogenous credit risk and liquidity measure. The S&P rating class index yield spread is used to proxy for credit risk, the ECB financial market liquidity indicator for liquidity. $\beta$ denotes the cointegration coefficient, $\alpha = (\alpha_b, \alpha_s)$ the coefficient vector of the error correction term, and $\Gamma_{b/s}$ the coefficient vector of the lagged bond (b) and CDS (s) premium changes with $\Gamma_b = (\Gamma_{b,b}, \Gamma_{b,s})$, $\Gamma_s = (\Gamma_{s,b}, \Gamma_{s,s})$. The top row of each panel displays the number of firms for which a) the augmented Dickey-Fuller test could not reject a unit root in the premia time series at the 10% significance level, b) the augmented Dickey-Fuller test could reject a unit root in the first differences at the 10% level, c) the Johansen test could not reject cointegration of the time series at the 10% level, d) the augmented Dickey-Fuller could reject a unit root in the residuals of the VECM at the 10% level. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms. The adjusted $R^2$ is given in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: All</th>
<th>Panel B: Investment Grade</th>
<th>Panel C: Subinvestment Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta bd$</td>
<td>$\Delta sl$</td>
<td>$\Delta bl$</td>
</tr>
<tr>
<td># Firms</td>
<td>145</td>
<td>142</td>
<td>140</td>
</tr>
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<td>$\beta$</td>
<td>-1.00***</td>
<td>-0.98***</td>
<td>-0.01*</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.88***</td>
<td>-0.43***</td>
<td>-0.01*</td>
</tr>
<tr>
<td>$\Gamma_b$</td>
<td>-0.77***</td>
<td>0.70***</td>
<td>-0.32***</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>0.64***</td>
<td>-0.86***</td>
<td>-0.03</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>0.03***</td>
<td>0.03***</td>
<td>0.01**</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-0.50***</td>
<td>-0.70***</td>
<td>-0.48***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>15.36</td>
<td>13.23</td>
<td>26.68</td>
</tr>
</tbody>
</table>
Table VII: Impact of Increasing and Decreasing Market-Wide Risk

The table presents the estimated coefficients for the VECM in increasing and decreasing credit risk phases. Increasing risk phases are defined as intervals with 4 consecutive weekly increases in the S&P yield spread for a rating class, decreasing risk phases analogously. The dependent variables are the premium changes, the explanatory variables are the vector error correction terms and the lagged premia changes. $\beta$ denotes the cointegration coefficient, $\alpha = (\alpha_b, \alpha_s)$ the coefficient vector of the error correction term, and $\Gamma_{b/s}$ the coefficient vector of the lagged bond (b) and CDS (s) premium changes with $\Gamma_{b} = (\Gamma_{b,b}, \Gamma_{b,s})$, $\Gamma_{s} = (\Gamma_{s,b}, \Gamma_{s,s})$. ***, **, and * denote significance at the 1%, 5%, and 10% level for a standard t-test across firms. The adjusted $R^2$ is given in percentage points.

<table>
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<tr>
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<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>$\Delta bd$</td>
<td>$\Delta dl$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-0.98^{***}$</td>
<td>$-0.99^{***}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.82^{***}</td>
<td>-1.40^{***}</td>
</tr>
<tr>
<td>$\Gamma_{b}$</td>
<td>-1.40^{***}</td>
<td>0.72^{***}</td>
</tr>
<tr>
<td>$\Gamma_{s}$</td>
<td>0.35^{***}</td>
<td>-1.56^{***}</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>15.47</td>
<td>11.42</td>
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</table>

Panel A: All

Panel B: Investment Grade

<table>
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<tr>
<th></th>
<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta bd$</td>
<td>$\Delta dl$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-0.99^{***}$</td>
<td>$-0.99^{***}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.14^{**}</td>
<td>0.07^{***}</td>
</tr>
<tr>
<td>$\Gamma_{b}$</td>
<td>-1.01^{***}</td>
<td>0.46^{***}</td>
</tr>
<tr>
<td>$\Gamma_{s}$</td>
<td>0.22^{***}</td>
<td>-0.77^{***}</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>7.25</td>
<td>6.52</td>
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Panel C: Subinvestment Grade

<table>
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<tr>
<th></th>
<th>Increasing Risk Phase</th>
<th>Decreasing Risk Phase</th>
</tr>
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<tr>
<td></td>
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<td>$\Delta dl$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-2.92^{***}$</td>
<td>$-4.74^{***}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-2.92^{***}</td>
<td>-0.07^{***}</td>
</tr>
<tr>
<td>$\Gamma_{b}$</td>
<td>-2.60^{***}</td>
<td>1.14^{***}</td>
</tr>
<tr>
<td>$\Gamma_{s}$</td>
<td>1.03^{***}</td>
<td>-1.91^{***}</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>23.09</td>
<td>17.17</td>
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</table>
Table VIII

Estimated Credit Risk, Liquidity, and Correlation Premia using CDS Ask or Bid Premia

The table presents the sample mean, standard deviation, and mean absolute difference between the credit risk, liquidity, and correlation premia estimated from CDS bid and ask premia simultaneously (column 2), using only CDS ask premia (column 3), and using only CDS bid premia (column 4). \( bd \) gives the pure credit risk, \( bl \) the pure liquidity, and \( bc \) the correlation component in the yield spread of a synthetical 5-year par bond. \( sd \) gives the pure credit risk, \( sl \) the pure liquidity, and \( sc \) the correlation component in the mid premium for a 5-year CDS contract. All values are in basis points.

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<th>Ask Only</th>
<th>Bid Only</th>
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<td>( bd )</td>
<td>44.38</td>
<td>44.15</td>
<td>44.78</td>
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<tr>
<td>Std. Dev.(( bd ))</td>
<td>83.46</td>
<td>84.76</td>
<td>84.99</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.48</td>
<td>0.55</td>
</tr>
<tr>
<td>( bl )</td>
<td>26.36</td>
<td>26.98</td>
<td>25.69</td>
</tr>
<tr>
<td>Std. Dev.(( bl ))</td>
<td>46.20</td>
<td>47.90</td>
<td>48.02</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>1.37</td>
<td>1.42</td>
</tr>
<tr>
<td>( bc )</td>
<td>3.59</td>
<td>3.48</td>
<td>3.69</td>
</tr>
<tr>
<td>Std. Dev.(( bc ))</td>
<td>12.04</td>
<td>12.90</td>
<td>13.02</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>( sd )</td>
<td>44.76</td>
<td>44.20</td>
<td>45.33</td>
</tr>
<tr>
<td>Std. Dev.(( sd ))</td>
<td>82.96</td>
<td>83.01</td>
<td>82.16</td>
</tr>
<tr>
<td>Mean Abs. Difference</td>
<td>–</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td>( sl )</td>
<td>1.94</td>
<td>1.50</td>
<td>2.34</td>
</tr>
<tr>
<td>Std. Dev.(( sl ))</td>
<td>10.74</td>
<td>8.25</td>
<td>8.29</td>
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<tr>
<td>Mean Abs. Difference</td>
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<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td>( sc )</td>
<td>0.41</td>
<td>1.01</td>
<td>0.28</td>
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<td>Std. Dev.(( sc ))</td>
<td>4.91</td>
<td>6.23</td>
<td>3.82</td>
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<tr>
<td>Mean Abs. Difference</td>
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<td>0.38</td>
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