Option Valuation: Theory and Empirical Evidence

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Summary: This paper reviews option valuation theory and the empirical evidence. The rapid growth of interest in option theory is probably due to the abundance of relevant applications in the financial marketplace. The precision of the option valuation models relies primarily on preference-free, enforceable arbitrage conditions. First, these arbitrage conditions are reviewed, and the related partial equilibrium hedging models are discussed. Next, the more general equilibrium, non-hedging models are briefly surveyed. Then, the differences in options on equity, debt, currency, and futures are mentioned, along with other applications of option theory.

Most of the empirical work testing the arbitrage boundaries has been related to equities, where the market data originated. Empirical tests of the boundaries and tests comparing various option models are reviewed for options on a variety of underlying assets. The estimation problems most relevant to option pricing are also discussed.

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Capital Market Equilibria. Ed. by G. Bamberg and K. Spremann

1. Introduction

The history of trading options is very long, and it is also rather checkered. Stories in the Bible describe both the terms and the exchange of options, as do historical accounts of Medieval and Renaissance periods. Usually, these tales attribute great risk and often much ruin to those foolish enough to trade options. No doubt, the wisdom of the times is reflected in the tales. However, today the wisdom of the times is greater, at least with respect to options. In the last fifteen years, we have learned to regard the trading, the pricing, and the uses of options as a precise science. Today, we know exactly why past predictions of the great risk involved with trading options were correct, but we have further learned how to use options to control risk. As we will see, the most widely used model for valuing options is the European model developed by Black and Scholes (1973). However, almost all options traded in the world market place are American options. The distinction between European and American options is somewhat misleading, because it has nothing to do with geography, but instead describes the differences between the exercise properties of two options. The European option can only be exercised at one specific point in time, the expiration date of the option. On the other hand, an American option can be exercised at any point in time between the origination date and the expiration date of the options.

This paper reviews the development of modern option pricing theory (section 2) and the results of the corresponding empirical tests (section 3). In section 2.1, we discuss the preference and distribution-free results. Here, bounds on the range of feasible prices are implied by arbitrage conditions. Subsection 2.2 discusses distributional assumptions and hedging models. This approach is popular with traders because hedging specifies a technique for reducing risk while making markets in options. The preference assumptions of non-hedging models, new option instruments, and applications of option theory to other financial valuation problems are discussed in the remaining subsections of the theoretical part. Section 3 starts with a discussion of the problems of testing model validity and market efficiency. Subsections 3.1 and 3.2 report results of boundary condition tests. These boundary tests avoid confounding joint hypotheses of both model validity and market efficiency. The empirical evidence on different option pricing models is discussed subsequently, and estimation problems relevant to option pricing are briefly reviewed.

2. Option Valuation Theory

2.1 Preference and Distribution-Free Results

The choice of whether to buy or sell one risky asset or another is an example of a decision problem under uncertainty. Expected utility theory has been developed and refined for many uncertainty problems since its advent during World War II. There, it drew attention for many important investment decisions, such as the inventory stock-out problem of supplying bullets to a machine-gun nest. The theory focuses on the tastes (or preferences) and beliefs (or probability distributions) of individuals, subject to a variety of possible constraints. In finance today, many decision problems under uncertainty are about optimal investment decisions, subject to the constraint of no margin, taxes, or being fully invested. Another major research topic in financial theory is asset valuation under uncertainty.

Most valuation models in modern financial economics, such as the capital asset pricing model (CAPM), the option pricing models (OPM), and the arbitrage pricing models (APM), impose restrictions on the individual's tastes (for example quadratic utility), or beliefs (normal or lognormal distributions) in order to simplify the problem. However, in option pricing theory, Merton (1973a) has shown that merely by assuming individuals which prefer more to less (i.e., are rational), arbitrage will place restrictions on the values of put and call options. This section discusses these arbitrage restrictions. The arbitrage results will be stated but not proven.

We assume that markets are perfect, which allows us to ignore short sale restrictions, margin requirements, transaction costs, taxes, differences in borrowing and lending rates, illiquidity, and price pressures. All of these effects can be introduced, some more easily than others. Interest rates can be random, but they must always be positive. The arbitrages will be between options differing only in time to expiration, options differing only in exercise price, or between options, the underlying stock, and default-free zero-coupon bonds.

2.1.1 Call Options

A call option gives its owner the right to buy a specified asset for a specified price within a specified time period. The specified price of the underlying asset is termed the exercise or strike price K .

The specified expiration date T of the option implies a time premium which is wasting. An option (put or call) differs from a futures contract where the holder must take delivery of the underlying asset at expiration, or buy back the future to avoid that obligation. By definition, the optionholder has the option to take delivery, and it is the theory of this option value which is the focus of much discussion. The following paragraphs begin the review of the valuation theory.

The value C of a call option will never be less than zero. This is because limited liability of the stock restricts the stock price S to be positive. Also, the value of a call will never be worth less than either zero or its exercisevalue $S-K$. Furthermore, the value of a European call will be greater than the exercise value by the difference between the exercise price and the present value of the exercise price. In fact, if dividends are paid over the life of the option and the call is dividend unprotected, the call value will be greater than the difference between the stock price and the present value of the exercise price, less the present values of any stock dividends to be paid prior to expiration. Finally, the call will never sell for more than the stock. These conditions can be expressed by the following inequality:

$$S \geq C \geq \max(0, S - Ke^{-rT} - \sum_j D_j e^{-rt_j}) \qquad (1)$$

where r is the riskless interest rate, and D_j denotes the amount of dividend paid at date t_j .

Arbitrage also forces restrictions on the relation between changes in the call prices as the exercise price changes. Given two calls which differ only in exercise price, the one with the lower exercise price, K_1 , must be at least as valuable as the other. Furthermore, the difference in value between these two calls must never be greater than the difference in their exercise prices. This can be expressed by the following inequality:

$$-1 \leq [C(K_2) - C(K_1)]/(K_2 - K_1) \leq 0 \ . \qquad (2)$$

Finally, given three calls (butterfly spreads) differing only in exercise prices, the value of the call with the middle exercise price can never be greater than a weighted average of the two extreme calls (convexity). This can be expressed as:

$$C(K_2) \leq w_1 C(K_1) + w_2 C(K_3) \ , \qquad (3)$$

where the weights (w_1 and w_2) are the ratios of the partial to the total difference in the exercise prices:

$$w_1 = \frac{K_3 - K_2}{K_3 - K_1}, \quad w_2 = \frac{K_2 - K_1}{K_3 - K_1} \tag{3a}$$

The time value of an American option is always positive. Given two calls which differ only in time to expiration, the one with more time to expiration must be at least as valuable as the other. When T_2 is greater than T_1, this relation can be expressed as follows:

$$C(T_2) \geq C(T_1) \ . \tag{4}$$

Since American calls can be exercised prior to their expiration date, arbitrage imposes restrictions on the optimal exercise policies. Merton (1973a) showed that an American call will sell for the same price as a European call when the underlying stock does not pay dividends. In this instance, the option to exercise early has no value. The logic is as follows: The owner of an American call can take one of three choices: (i) hold it; (ii) exercise it; or (iii) sell it. Since the premium above (i.e., exercise value) is always positive prior to expiration, selling the call dominates exercising it, except when the underlying stock pays dividends. Then, it may be rational to give up the premium above parity to get the dividend. Thus, an American call should only be exercised just before an ex-dividend date or at expiration. If it is ever optimal to exercise a particular call option, then all other calls with either smaller exercise prices or less time to expiration should also be exercised at that time. However, it will never be optimal to exercise an American call early if the present value of the dividends to be paid over the life of the call are less than the possible interest that can be earned on the exercise price during the same time period.

2.1.2 Put Options

A put option gives its owner the right to sell the specified asset for a specified price within a specified time period. Thus, a put seems to be the opposite of a call, since semantically, the right to sell is the opposite of the right to buy. However, in option valuation theory, such simple reasoning can lead to confusion. Both the right to buy and the right to sell have unique value, which we will see depend on the allowable range of the underlying asset price. The following paragraphs discuss this theory of value for puts.

Similar to call options, arbitrage restrictions exist for unprotected American put options. An American put will never sell for less than either zero or the difference between the exercise price and the stock price, $K-S$. The maximum value a put can achieve is its exercise price, because of the limited liability of the stock. The relations can be summarized by the following inequality:

$$K \geq P \geq \max(0, K-S) \quad , \tag{5}$$

where P denotes the put value.

Arbitrage also forces restrictions on the relation between changes in put prices for different exercise prices. Given two puts which differ only in exercise price, the one with the higher exercise price must be at least as valuable as the other. Furthermore, the difference in value between these two puts must never be greater than the difference in their exercise prices. This can be expressed by the following inequality:

$$1 \geq [P(K_2) - P(K_1)] \,/\, (K_2 - K_1) \geq 0 \tag{6}$$

Finally, given three puts (butterfly spread) differing only in exercise price, the value of the put with the middle exercise price can never be greater than a specific weighted average of the values of the two extreme puts. This can be expressed as follows:

$$P(K_2) \leq w_1 P(K_1) + w_2 P(K_3) \tag{7}$$

where the weights (3a) are ratios of the partial to the total differences in the exercise prices (convexity).

Since the time value of an American put is always positive, given two puts which differ only in time to expiration, the one with more life remaining will be more valuable. If T_2 is greater than T_1, this can be expressed as:

$$P(T_2) \geq P(T_1) \tag{8}$$

However, for a European put, more time to expiration can sometimes reduce value. The intuition is, if it is optimal for the putholder to exercise early, then more time to expiration only prolongs the agony of not being able to exercise an option. Merton (1973a) also demonstrated that an American put always has a positive probability of premature exercise. This occurs because the limited liability of the stock bounds the stock price below at zero. Thus, for some (low) stock price, it is better to take the exercise proceeds and invest them at the risk-free rate rather than wait for the additional option value. At this critical stock price where the put is exercised early, the premium above parity has gone to zero and the put is worth its exercise value. Dividends paid on the underlying stock increase put values because theyreduce the stock price, and thus, dividends reduce the probability of early exercise. In fact, the put should never be exercised prematurely if the present value of the dividends to be paid prior to expiration of the put exceed the present value of interest that could be earned on the exercise price during the same time period. Finally, if it is ever optimal to exercise a particular put, then all other puts with either a larger exercise price or less time to expiration should also be exercised at that time.

2.1.3 Relations Between Puts and Calls

Converters, reverse converters, and box spreaders make a business of converting calls into puts, puts into calls, and calls, puts, and the underlying stock into risk-free bonds. These conversions or replications hold exactly for European options, and the relation between these securities is termed put-call parity. The following arbitrage condition describes this relation for European options having the same exercise price and expiration date:

$$P = C - S + Ke^{-rT} . \qquad (9)$$

This implies that a bought put can be duplicated by a bought call, short stock, and lending the present value of the exercise price.

American options complicate these conversion and replication strategies. The package of securities necessary to replicate the American option's countable but infinite set of exercise contingencies, is itself infinite (see Geske and Johnson, 1984) . The simple equality of European put-call parity does not hold, and only a pair of inequalities hold for either payout-protected American options or for options written on non-dividend paying stock:

$$C - S + K \geq P \geq C - S + Ke^{-rT} \qquad (10)$$

When dividends are paid and the option is not payout-protected, these bounds must include the dividends. Uncertainty about interest rates and about dividends further complicate these bounds. Transactions costs, margin requirement, and differential borrowing and lending rates widen the inequality bands bounding the American put value.

2.1.4 Additional Arbitrage Restrictions

In all of the above arbitrage relations, no assumption about the distribution of stock price changes was necessary. However, in order to discuss even weak restrictions with respect to stock price, it is necessary to assume that the distribution of stock price changes is independent of the level of the stock price. Although this is generally not valid, as even casual empiricism would reveal, it may not be a bad assumption. Given this assumption, the value of one call can never be greater than the value of an otherwise identical call with a higher stock price. If S_2 is greater than S_1, this can be expressed as:

$$C(S_2) \geq C(S_1) \tag{11}$$

Also, given three calls which are identical, except for different stock prices, the value of the call on the middle stock price can never be greater than a weighted average of the values of the calls on the two extreme stock prices.

$$C(S_2) \leq w_1 C(S_1) + w_2 C(S_3) \tag{12}$$

where the weights (comp. (3a)) are ratios of the partial to the total differences in the stock prices (convexity).

2.2 Distributional Assumptions and Hedging Models

2.2.1 Hedge Portfolios

In the last section, we were able to learn something about rational option pricing by studying the restrictions placed on option values by arbitrage. Most of the arbitrage results did not require very specific assumptions about individual preferences (i.e., utility) or beliefs (i.e., distribution of stock price changes). It was only necessary to assume that investors are rational, and for the arbitrage restrictions with respect to the stock price, that the distribution of stock price changes be independent of the level of the stock price. In order to be more specific about the price process for options in a securities market where individuals allocate their wealth to select optimal investments, we need to be more specific about either their preferences, their beliefs, or both. Here, we will make stronger assumptions about investors' beliefs regarding the distribution of stock price changes in order to present the hedging models for option values. The European option models will be discussed first because they are simpler than the American option hedging models.

First, assume that investors believe that the distribution of stock price changes follows a lognormal (proportional) diffusion process. The lognormal is empirically preferred to a normal (absolute) diffusion because the probability of price changes appears to be a constant proportion of the level of the stock price rather than a constant absolute amount. Now, if investors can trade continuously for this price process, and if the option's price evolution is derived entirely from stock price changes so that the option and the stock are perfectly correlated, then investors can form perfect, self-financing, riskless hedges between the option and underlying stock. This hedge portfolio contains two risky securities, the option and the stock. Earlier work in finance taught us that in order to efficiently diversify risk in a two-security portfolio, the higher the correlation among the two securities, the better the diversification. If the two risky securities are perfectly correlated, then by buying one and selling (writing) the other in the correct proportions all the risk can be eliminated.

In order to demonstrate this concept, consider the following hedge portfolio, H, consisting of n_S shares of stock and n_C calls:

$$H = n_S S + n_C C \ . \tag{13}$$

Now, if the stock price changes, inducing a change in the call price, and assuming all else is (approximately) constant, what proportion of shares of stock to calls can the investor hold so that the value of the hedge will not change? To derive this "hedge ratio", set the change in the value of the hedge, dH, induced by changes in the stock price, dS, and call price, dC, to zero, and solve for the number of calls to write (buy) per share of stock long (short). This yields

$$dH = n_S \, dS + n_C \, dC = 0 \ . \tag{14}$$

If we take $n_S = 1$ we have

$$n_C = -1/(dC/dS)$$

where dC/dS is the hedge ratio. This simple example is illustrative, but is only approximate because as the stock price and call price change, so must the ratio of calls per share of stock in order to remain risk-free. Furthermore, when the investor sells out of one position in order to obtain the required new riskless hedge ratio, the hedging transaction must be self-financing. Fortunately, this is exactly what happens. In order to make this hedge an enforceable arbitrage, a third security, riskless bonds, should be added to the hedge portfolio. When riskless bonds are included, it is immediately evident that the option investment can be exactly replicated by a levered position in the underlying stock. This replication is the forcing mechanism of the arbitrage between the option, the stock, and riskless bonds. The riskless bonds also facilitate the self-financing.

The solution to the European call option pricing problem will depend on the assumption made to describe the distribution of stock price changes, dS . Before stating the various assumptions leading to different hedging models, consider the notable implications of the hedging approach. First, any investor who is perfectly hedged does not care whether the stock price goes up or down, since the value of the hedge is independent of the direction of stock price movements. This intuition implies that the option value may be independent of the expected return on the stock. Second, the arbitrage conditions necessary to maintain the hedge only require that investors be greedy, but not risk-averse. Investors could prefer risk and still the above riskless arbitrage portfolio would place restrictions on the relation between the option price and the stock price. Third, if the stock price is the only variable assumed to be random in the hedge, then the call value will only depend on one random variable. Fourth, the ability to create riskless hedges does not imply that investments in call options are riskless, or that investors only earn the risk-free rate on their investment. In fact, if an investor correctly feels that an option is over or underpriced, the proper establishment of this arbitrage portfolio will yield returns in excess of the risk-free rate, at no risk.

Now, return to the consideration of the relevant assumption for the distribution of stock price changes, dS . The origin of the alternatives discussed here stems primarily from empirical observations of stock price movements. The models are simple attempts to capture the observed phenomena. There are two main differences in these models of stock price changes:

(i) whether the price changes are continuous
(ii) whether the variance of price changes is constant.

The seminal model in modern option pricing theory (Black and Scholes 1973) assumes that the stock price changes are continuous with a constant variance of price changes. The other models vary these assumptions in an attempt to capture differences in the way information induces stock price changes. Several models are catalogued in Table 1. Some motivation of the assumptions is contained in Sec. 2.2.3. The equations resulting from the models are given in the Appendix.

Table 1: Assumptions of Models for Pricing Options

Model	Price Change	Variance	Motion Equation
1. Diffusion (Black-Scholes)	Continuous	Constant	$dS = \mu Sdt + vSdz$, where dz is the increment of a standard Gauss-Wiener process and μ is the expected instantaneous rate of return
2. Binomial (Cox, Ross, Rubinstein)	Discrete	Constant	$S(t+1) - S(t) = f(u,d)S(t)$
3. CEV (Cox-Ross)	Continuous	Changing	$dS = \mu Sdt + vS^{\rho}dz$
4. Compound (Geske)	Continuous	Changing	$dS = \mu Sdt + (\partial S/\partial V)(V/S)v'Sdz$
5. Displaced (Rubinstein)	Continuous	Changing	$dS = \mu Sdt + v'Sdz$
6. Jump (Cox-Ross)	Discontinuous	Constant	$dS = (\mu-\lambda k)Sdt + Sdq$ (dq = increment of a Poisson process with parameter λ)
7. Diffusion-Jump (Merton)	Continuous/ Discontinuous	Constant	$dS = (\mu-\lambda k)Sdt + vSdz + Sdq$

2.2.2 The Classical Black-Scholes Model

In the partial equilibrium diffusion model of Black and Scholes (1973), the hedge between the stock, the option, and the riskless bond is maintained continuously. The arbitrage relation between the call and its replication by a unique position in the stock and riskless bond implies that the resultant valuation equation for the option does not depend on whether the stock price is in equilibrium. In other words, all information may not be reflected in the stock price which therefore may be over or undervalued, and still the option valuation equation must hold. The Black-Scholes equation is only for European options, and is given by:

$$C = SN(d_1) - Ke^{-rT} N(d_2)$$

where

S = stock price at current date

K = exercise price

T = time to expiration

v^2 = variance of the stock's rates of return

r = riskless interest rate

$N(\cdot)$ = the cumulative standard normal distribution

$$d_1 = \frac{\ln(\frac{S}{K}) + rT + \frac{1}{2} v^2 T}{v\sqrt{T}}$$

$$d_2 = d_1 - v\sqrt{T}$$

First, note that the expected return on the stock does not directly enter this equation. It only enters indirectly through the stock price, which is observable. As predicted in hedging models, the expected direction of stock price movements is not important to the value of the option. This is the primary reason why hedging models are so accurate relative to other financial valuation models, and probably the reason they have gathered the attention of so many academics and practitioners.

Although this formula appears complex, it has at least two intuitive interpretations. The first is that it represents the present value of the probability of receiving the exercise value, if and only if, $S(\tau)$, the stock price at expiration, is greater than K, the exercise price. The first term, $SN(d_1)$, is the present value of receiving the stock, and the second term, $-Ke^{-rT}N(d_2)$, is the present value of paying the exercise price, if and only if the option is exercised. Another interpretation is based on the arbitrage idea that the call can be replicated by a levered position in the stock. Then the first term is the amount invested in the stock, and the second term is the amount borrowed.

2.2.3 A Brief Description of Other Option Valuation Models

The binomial hedging model (Cox, Ross, Rubinstein, 1979) is based on a discrete process where the stock price can move to only one of two positions in a finite time interval. The size of the up and down (u and d) movements is a constant proportion of the stock price, and this implies a constant variance of stock price movements. The primary advantage of the binomial model is simplicity. In the limit, as the discrete time interval is reduced to zero, the binomial model can be made to converge to the Black-Scholes model. In addition, the binomial process is flexible, and depending on how the limits are taken and what is assumed about the up and down movements, the binomial process can incorporate the changing variance and jump process models. While this flexibility is useful pedagogically, empirically, the limiting parameters of each model must be estimated and then converted into the proper up and down jumps.

CEV means constant elasticity of variance models (Cox-Ross, 1976b). In these models, the stock price process is continuous, and the variance of stock price changes is dependent on the level of the stock price. The volatility of the stock's rate of return is $\hat{v} = vS^{\rho-1}$. The elasticity of this olatility with respect to the stock price, $(\partial\hat{v}/\partial S)(S/\hat{v})$, is a constant, namely $\rho-1$. For these processes, if $\rho < 1$, the variance of the rate of return varies inversely with the stock price. This property has been empirically established by many researchers. If $\rho > 1$, the opposite (direct) relation holds between the volatility and the stock price. When $\rho = 1$, the CEV corresponds to the Black-Scholes constant variance model. Thus, the CEV model is also flexible since it can handle a variety of relations between the stock price and the volatility, and it contains the Black-Scholes model as a special case.

The compound option model (Geske, 1979a) offers an economic rationale for the inverse relation observed between the volatility of stock price changes and the stock price. It is a capital structure argument, based on the observation that as the stock price changes with a fixed amount of corporate debt outstanding, the firm's leverage ratio will vary inversely with the stock price. This will induce the observed inverse relation between stock return volatility and stock price. To keep the problem simple, the firm is assumed to consist of only pure discount bonds maturing at T and stock. The stock is then considered to be an option on the assets of the firm, and an option on the stock is an option on an option.

In the compound option model, the volatility of the stock rate of return is nonstationary, changing randomly as the leverage changes. Although the resultant formula appears complex (see Appendix for definitions of terms), it has in intuitive interpretation. The first two terms of the formula, $VN_2(h, k; \rho) - Me^{-rT}N_2(h', k'; \rho)$, represent the present value of receiving the stock if the stock price at the option expiration date T is greater than the exercise price ($S(T) > K$, or if $V > \bar{V}$), just as in the Black-Scholes formula, only now the stock is itself an option on the firm. The third term in the formula is identical to the Black-Scholes second term $(-Ke^{-rT}N(h))$, and represents the present value of paying the exercise price, if and only if $V > \bar{V}$ (or $S(T) > K$). The compound option model contains the Black-Scholes model as a special case when there is no effective leverage in the firm (i.e., $M = 0$ or $T_M = \infty$).

The displaced diffusion model (Rubinstein, 1983) is a third changing variance model for which the simple Black-Scholes equation is again a special case. This model is motivated from the asset side of the balance sheet, while the compound option model is motivated by the liabilities. In the compound option model, the changing liabilities of stocks and bonds induces the nonstationarity in the stocks' variance. In the displaced diffusion model, a portfolio of risky and riskless assets is the source of the changing variance. To keep the problem simple, the firm is assumed to own two types of assets, one risky and one riskless. If the value of the firm rises at greater than the riskless rate, it must be due to the return on the risky asset. This shifts the portfolio composition toward the risky asset, and since the volatility of both assets is assumed constant, the volatility of the firm and also of the stock will rise. Thus, there is a direct relation between the stock price and its volatility, which is opposite to the relation in the compound option model.

In diffusion models, price changes are small and continuous, whereas empirically, we sometimes notice large and discontinuous price changes. This difference in price changes can be conceptualized as alternative models of the rate of arrival of unexpected, important information. An example would be the discovery of a new mine or a tender offer for the firm. To construct a hedging model of discontinuous price changes, the discontinuity must be deterministic. This is one example of a pure jump process, where the price is constant between jumps, and very infrequently a jump of constant amplitude occurs. The resulting jump model (Cox-Ross, 1976b) has two terms as in the Black-Scholes equation, and the interpretation of them is similar to before. However, the probability terms indicative of whether the stock price is greater than the exercise price are complementary Poisson, rather than cumulative normal. Although this model allows for the empirically observed discontinuous price changes, the assumption that in between infrequent jumps the price remains constant, and at the jumps the size of the price move is constant seems unrealistic.

The diffusion-jump model (Merton, 1976a) is an attempt to incorporate a more realistic distribution than the pure jump process. If the amplitude of the jump is allowed to be random, and if between jumps small random (diffusion) price changes are allowed, then the resultant model will no longer be a pure hedging model. No arbitrage portfolio can be constructed to remove this generalized uncertainty. However, if the jumps of individual stocks are assumed to be uncorrelated with the market, then all of the jump risk is nonsystematic and thus diversifiable. The intuition of the solution to this problem is that a hedger must hold a well-diversified portfolio to eliminate the jump risk, and then continuously adjust the portfolio to hedge the diffusion risk. Thus, the resulting formula contains the sum of infinitely many Black-Scholes type calls weighted by the Poisson probabilities reflecting the likelihood of the jumps. The variance of the jump-diffusion differs from the pure diffusion, but it is stationary.

2.2.4 Analytic Models for American Calls and Puts

All of the above hedging models listed in Table 1 were developed for valuing European options, but listed options are American and can be optimally exercised at any time prior to expiration. As previously discussed, puts always have a positive probability of early exercise, while calls may only be exercised just before a dividend payment or at expiration. Many approximations exist which accurately price American options. In discrete time, the binomial model can be modified to check for exercise at all relevant instants. In the case of listed call options on stock which currently have at most a nine-month life in the U.S. (in Germany nine and one half months), there would be a maximum of three dividends and one expiration, or four exercise times during the life or the option. For currently listed puts, approximately 150 exercise checks are necessary for penny accuracy (Geske-Shastri, 1985). In continuous time, when approximating the partial differential equation subject to the relevant exercise boundaries, a similar number of critical stock price computations are required for accurate valuation of both puts and calls. However, when evaluating an analytic solution to the continuous time problem, only three or four exercise checks are necessary for accuracy.

These analytic models of American calls (Roll, 1977, Geske, 1979b, and Whaley, 1981) and puts (Geske-Johnson, 1984) are based on the compound option model. At each relevant exercise instant, the investor has an option on an option, and the choice is either to exercise or to take the next option. The solutions are differentiable, and thus provide analytic formulae for hedge ratios and other sensitivities. Furthermore, this compound option approach can be used to value American options on currencies and futures, where both calls and puts have a positive probability of premature exercise at every instant. Shastri-Tandon (1984a), Whaley (1984a), Bodurtha-Courtadon (1984) model American options on currencies, while Shastri-Tandon (1984b), Whaley (1984b), Brenner, Courtadon, Subrahmanyan(1984), Ramaswamy and Sundaresan (1984), and Ball-Torous (1985b)model American options on futures.

The hedging models based on arbitrage provide unique insights about preference-free valuation. In fact, since preferences do not enter the problem, the solutions should be consistent with any set of preferences.[1] Thus, if we assume risk neutrality, we should be able to discount the expected value of the option at expiration by the riskless interest rate to find the option's current value. In order to compute the expected value of the option at expiration, if we can find the terminal distribution of stock price changes conditional on the current stock price, then we can solve the option valuation problem by conventional discounting of cash flows. Cox-Ross (1976b) demonstrated this solution technique. However, if we cannot create self-financing, riskless hedges, then preferences will enter the valuation process. If trading is not continuous, possibly because of transaction costs, or if other sources of risk enter the option problem, possibly through random interest rates or payouts, or if the jumps are correlated with the market, then perfect hedging may not be feasible, and preferences may enter the problem. The next section discusses preference-based option models.

2.3 Preference Assumptions and Non-Hedging Models

Prior to the growth of interest in option models, most valuation theory in financial economic approached the valuation problem from the demand, or preference side. In the recent tradition of solving choice problems under uncertainty by assuming individuals maximize expected utility of terminal wealth, much effort has been placed on investigating the conditions where individual demand equations can be aggregated to arrive at a market clearing valuation equation. The work of Wilson (1968) and Rubinstein (1974) documents the sets of assumptions on preferences (i.e., utility functions) and beliefs (i.e., distributions) which will allow aggregation yielding equilibrium valuation equations. The difficulty with most of these solutions is that they involve the estimation of parameters such as the

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1) See Harrison and Kreps (1979) and Kreps (1981) for details regarding preference requirement in arbitrage pricing.

expected return on the market, or a market risk aversion parameter where the measurement error is large.

Rubinstein (1976) was the first to demonstrate that the continuous trading assumption could be relaxed in a preference-based setting, and yet a preference-free option model would result. If utility functions are of the constant proportional risk averse (CPRA) family, and stock returns are lognormally distributed, then the resultant option valuation equation is identical to Black-Scholes. At first, this seemed to be a paradox, since riskless hedging appeared to be the key ingredient necessary for a valuation expression independent of an expected return. However, recall that investors who possess the class of constant absolute and constant proportional risk averse utility functions behave myopically (i.e.,are shortsighted). Myopia implies that investors today act as if they will not value the opportunity to revise their portfolios tomorrow (Mossin, 1968), even if they ultimately do revise tomorrow.

Brennan (1979) extended these preference-based results by showing that with constant absolute risk averse utility functions, and normal distributions of security returns, risk neutral valuation results also occur. These risk neutral valuation relations may provide valuable insights to our understanding of more complex option valuation problems where riskless hedging does not appear feasible. Rubinstein's (1976) result is based on a familiar "arbitrage" condition termed the "law of one price". The idea is that any two securities with identical cash flows in every state must sell for the same price. Brennan's (1979) basic valuation equation follows from the first order conditions of the portfolio optimization problem for the representative investor. Egle and Trautmann (1981) confirmed these results by using an alternative, more basic, "no-arbitrage" condition.

2.4 New Option Instruments

Since the advent of the equity option market in April, 1973, new options have been proposed and listed on many other instruments. Although some of the proposals are still pending by the Securities and Exchange Commission (SEC), today, options are listed on currencies, on stock indices, on debt instruments and on futures trading on these underlying instruments. Many of the earlier option results will apply to these new contracts, but there are some differences. For example, call options on futures and currencies will always possess a positive probability of premature exercise, and like the American put, will be more difficult to value. The empirical investigation of these new option instruments is just beginning, and some results will be presented later in this paper.

The concept of early exercise for an American call on a future can be seen immediately by considering a European call. Black (1976) showed that since arbitrage relates the futures price, F, to the current spot price S by the riskless cost of carry (i.e., $S = Fe^{-rT}$), the Black-Scholes European option pricing model will apply to European futures options simply by substituting Fe^{-rT} for the stock price S in equation (15). Now, as the futures price becomes large relative to the exercise price, the value of a European call on a future converges to $(F-K)e^{-rT}$ as a lower bound. Since the American call can be exercised immediately for $(F-K)$, early exercise is always possible. The reasoning is similar for American currency options, where the differential interest rate between domestic and foreign countries is similar to a stock paying a continuous dividend. Since call options on dividend-paying stocks can be exercised early, American call options on currencies will be more valuable than European calls.

Options on government securities (i.e., debt options) are more difficult to value because of the interest rate sensitivity of the fixed payout, and because the bond price converges to the amount of the terminal payout (i.e, face value). The fact that each fixed (cf., semiannual) payout is sensitive to a different interest rate implies that debt option values depend on many correlated random variables. Originally, most papers in this area assumed a specific stochastic process for the interest rate(s) which determined bond prices. For example, Brennan-Schwartz (1982) derived an equilibrium model of the term structure which implied equilibrium bond prices, and then options were priced off these bond prices. Geske (1981) pointed out that a hedging approach between the optioned bond, a financing bond, and the option might yield a better model because bond market prices could be used directly, regardless of whether the bond price was an equilibrium price.

While Brennan-Schwartz' equilibrium approach and Geske's hedging approach both required two random variables, Courtadon (1982) and Cox, Ingersoll, Ross (1985) presented a model based on only one random interest rate. Ball and Torous (1983) assumed a stochastic process for the bond price (Brownian bridge) which accommodated its convergence to a terminal value. The empirical work is slowly accumulating, while the theoretical work on this difficult problem continues.

2.5 Applications of Option Theory

As was first noted by Black-Scholes (1973) and Merton (1973a), option theory can be used to value corporate securities. Merton (1974) showed that the stock and pure discount bonds of a firm that has no other payouts can be valued directly by application of option theory. This leads to insights about the risk structure of interest rates. Black-Cox (1976), Geske (1977), and Cox-Ross (1976b) extended the analysis to consider junior debt, securities of arbitrary maturity, and general corporate payouts. Ingersoll (1976) and Brennan and Schwartz (1977) applied the theory to problems of dual funds and callable, convertible debt and preferred stock. Option theory has also been applied to interpretations of the firm's financing and investment decisions by Galai and Masulis (1976), insurance concepts, such as deposit insurance by Merton (1977b) portfolio insurance by Leland (1980) to warrant valuation (Galai-Schneller, 1978), and Emanuel (1983), leasing problems, rights offerings, and employee stock options (Smith-Zimmerman, 1976). The list of applications is already long and still growing. The empirical evidence collected to date on a variety of option concepts is the topic of the next section.

3. Empirical Tests of Option Valuation

The primary goal of this chapter is to present the main results of the empirical work in option valuation. After a discussion of the problems of testing model validity and market efficiency, the analysis concentrates on the empirical examination of the theoretical results discussed in Section 2. To begin, a few useful terms will be defined and explained:

1. *Value vs. Price:* The distinction is the market prices an asset while a model values the asset. An asset will be considered underpriced (overpriced) when the market price is below (above) the model value. Underpriced (overpriced) assets are considered to be good buys (sells).
2. *Efficient market:* A market (or a set of markets) will be termed efficient if no single trader can consistently make above-normal,

risk-adjusted profits on an after-transaction cost and after-tax basis.

3. *Synchronous markets:* Synchronous markets are markets in which (1) trading in related assets takes place simultaneously in time (trading synchronization) and, (2) the technology for registration of trades accurately reflects the transaction time and the time the price information is made available to market participants (data sychronization). Market synchronization is required in order to test for arbitrage profits, although it is not a necessary condition for market efficiency.

4. *Above-normal profits:* For a riskless portfolio strategy (e.g., buying one option and selling the appropriate number of underlying shares), above-normal profits mean profits in excess of the risk-free rate of interest (i.e., on a government bond with the same maturity as the strategy). An uncertain yield is considered above-normal, if, after adjusting for risk, it is in excess of the expected return. The expected return is determined by an asset pricing model (c.f., the capital-asset-pricing model (CAPM) or the arbitrage-pricing model (APM)).

5. *Ex-post tests:* Based on information at time t, a trading strategy is devised and, by assumption, a position is established based on prices at time t. The position will be liquidated one period later at $t+1$, based on prices available at that time. Such a test procedure ignores the time lag in acting upon a profit opportunity indicated by the date, t, mispricing signal.

6. *Ex-ante tests:* Based on information at time t, a trading strategy is devised, but the position is established at time $t+1$ at prices that are unknown at time t. The position will be liquidated one period later at $t+2$, based on prices available at that time.

The statistical inference based on empirical studies of option valuation is complicated by the fact that usually some or all of the hypotheses about model validity (and its correct parameter estimation), market efficiency, market synchronization and data accuracy are tested jointly. In order to gain clearer insights from these confounding joint tests, empirical analysis of option valuation can be classified in the following way:

1. Test of boundary conditions between an individual option and the underlying asset.
2. Tests of boundary conditions between different options and the underlying asset.
3. Tests of absolute price levels for options.

From Section 2, it is clear that most boundary conditions are model independent and, therefore, the corresponding tests are more rigorous than those of the third class. But even boundary condition tests are joint tests of market efficiency, market synchronization, and data accuracy if market synchronization cannot be guaranteed, and the tests are conducted as ex-post tests. Given accurate data and correct risk adjustment, above-normal profits from an ex-post test may indicate either market inefficiency or nonsynchronous markets. It is a definite indication of market inefficiency only if above-normal profits are revealed from an ex-ante test. Table 2 summarizes the expected results for the different permutations of the joint hypotheses regarding model validity, market efficiency, and market synchronization, given accurate (market) data.

Empirical tests of option valuation models may be further complicated by market imperfections. Besides ignoring commissions costs and taxes, most option models also ignore the bid-ask spread. The latter should be considered in an efficiency test, since a listed security is generally bought at its (higher) ask price and sold at its (lower) bid price. The observed bid-ask spread is the most important component of the transaction costs, whenever market efficiency is measured in terms of a lowest cost trader, such as a market maker on the trading floor paying negligible commissions and no brokerage fees (c.f.,Phillips and Smith, 1980). Other sources of problems are insufficient depth (i.e., number of option contracts traded at a given price) or the virtual illiquidity of some option contracts and markets (out-of-the-money options near expiration or the German stock option market before the 1983 changes in trading rules), or finally, the discreteness of trading with minimum price changes will affect the results (e.g., in the U.S. markets, one-eighth $ for options, and in the German stock option markets, one-twentieth DM for stocks and options).

Table 2

Joint Hypotheses and Expected Results

(Adapted from Galai, 1983)

Model is Correct and its Parameters are Correctly Estimated	Option Market is Efficient	Markets are Synchronous	Expected Results
Yes	Yes	Yes	Good predictions of option prices by the option pricing model for given stock prices. But, there is not way to make above-normal profits.
Yes	Yes	No	"Paper" ex-post profits, but no above-normal profits can be expected ex-ante.
Yes	No	Yes	Ex-ante above-normal profits are expected. No alternative model is expected to yield more.
Yes	No	No	Both "paper" and realized profits are expected.
No	Yes	Yes	Less than (risk-adjusted) expected profits, though normal.
No	Yes	No	Less than (risk-adjusted) expected profits, may be ex-post profits.
No	No	Yes	Potential for above-normal profits, but an alternative model may yield more.
No	No	No	Poor predictions of option prices; an alternative model may show higher ex-post and ex-ante profits.

3.1 Test of Boundary Conditions Among an Individual Equity Option and the Underlying Stock

Rational boundaries for the price of a simple call or put option are described by relations (1) and (5), respectively. Of these upper and lower boundary conditions, only the lower boundary condition for a call option has been empirically tested. Galai (1978) and Bhattacharya (1983) performed tests of dividend unprotected call options traded on the Chicago Board Options Exchange (CBOE) during the exchange's first eight months of operation (April 1973 to November 1973), and during the period August 1976 to June 1977, respectively. While Galai's (1978) early data ignores the depth of the market, the bid-ask spread, and transactions costs, Bhattacharya's (1983) better data does closely approximate the market's operational constraints. Both studies consider the time lag in acting upon a profit opportunity indicated by a mispricing signal. That is, market efficiency is tested by means of Galai's ex-ante test.

Table 3 presents the observed frequency and average size of three types of call option lower bound violations defined in the following equations characterizing (1) immediate exercise, (2) European lower bound, and (3) pseudo-American lower bound, respectively:

$$\varepsilon_1 = S - K - C \leq 0 \tag{16}$$

$$\varepsilon_2 = S - Ke^{-rT} - \sum_{j=1}^{n} D_j e^{-rt_j} - C \leq 0 \tag{17}$$

$$\varepsilon_3 = \max_{i \in I}(S - Ke^{-rt_i} - \sum_{j<i} D_j e^{-rt_j}) - C \leq 0 , \tag{18}$$

where $I = \{1, \ldots, n+1\}$, $t_0 = 0$ (the current date), $t_{n+1} = T$, and D_j denotes the amount of dividend paid at date t_j $(j = 1, 2, \ldots, n)$.

Table 3

Tests of Call Option Boundary Violations

(Results of Bhattacharya, 1983)

	Immediate exercise test $H_0 : \varepsilon_1 \leq 0$	European lower bound test $H_0 : \varepsilon_2 \leq 0$	Pseudo-American lower bound test $H_0 : \varepsilon_3 \leq 0$
Sample size	86,137	54,735	32,432
Mispricing frequency	1,120	1,304	442
Average mispricing per contract ($)	12.57	9.88	10.85
Number of positions executed	759	670	139
Average profit per contract ($)			
• for a zero transaction cost trader	4.91	5.17	8.20
• for a member of the NYSE	- 0.59	- 6.98	- 8.26
• for a member of the CBOE	- 7.00	-13.14	- 8.63

The first type of mispricing occurs if the immediate exercise value of the option is greater than its current market value (i.e., $\varepsilon_1 > 0$). The second type of mispricing takes into consideration the European lower bound for a dividend unprotected call option that is expected to be held to its maturity. That is, if $\varepsilon_2 > 0$, then the European call dominance condition is violated. Finally, if the expression defined as ε_3 is positive, then the pseudo-American lower bound is violated. As distinguished from the European lower bound, this lower bound considers the possibility of exercising the unprotected American call at any time t_j, just before the stock goes ex-dividend.

A comparison of Galai's and Bhattacharya's ex-post test results shows that the relative mispricing frequencies (mispricing frequency divided by sample size) observed in both studies are similar. The average dollar mispricing magnitudes per contract, however, are significantly smaller in the study of Bhattacharya (1983). This result is due to the consideration of bid-ask spreads in the calculation of the mispricing magnitude. A second reason for the smaller ex-post inefficiency in the later sample period might be "learning" as CBOE traders became more experienced. On the other hand, both studies have the following in common: mispricing occurs most frequently for deep-in-the-money options with a short time to maturity. This leads Bhattacharya (1983, p. 177) to conclude that if these options were held by (average) investors, "they may have sold the calls at a discount so as to avoid having to exercise them and incur a round-trip transaction cost in the stock." Thus, some of the mispricing may be due to the existence of different transaction costs for different market participants.

More importantly, the last three rows of Table 3 demonstrate that only in the zero transaction cost case, the observed mispricing could have been translated into above-normal profits on an average basis. Furthermore, by trying to simulate a trading strategy based on mispricing signals, Bhattacharya (1983), like Galai (1978) for his earlier sample period, observes a "substantial reduction in the mean profit opportunities coupled with their transformation into uncertain outcomes" (Galai 1978, p. 209). In summary, for the largest option market in the world, no evidence contrary to option market efficiency was found when transaction costs were taken into account.

In a recent paper, Trautmann (1985a) reports violations of the European lower bound for call options traded on the Frankfurt Option Exchange (FOE) during the first eighteen months after the 1983 trading rule changes (April 1983 to September 1984). Compared with the European lower bound test in Table 3, in this study the violation is defined by

$$\varepsilon_2' = \varepsilon_2 + \left(\sum_{j=1}^{n} D_j\right) e^{-rT} \qquad (19)$$

because stock options traded in Germany are (partially) payout protected (i.e., similar to Over-the-Counter (OTC) options in the United States). That is, when a stock pays a dividend, the exercise price of the option is reduced by the amount of the dividend. Also, when a company issues rights to purchase additional shares of stock at less than the market price, the exercise price is reduced by the market price of the right, if the rights expire before the options.

There were 891 violations of the bound, or about 1.4 % of the sample size of 63,391, when the transaction costs of the lowest cost trader were taken into account. The average mispricing magnitude was DM 59.96 per contract (comprising rights to 50 shares in the case of German stocks). When a strategy based on observed mispricing was adopted, the resulting executed positions averages losses of DM 60.58 net of the transactions costs for the lowest cost trader (e.g., the floor broker). Recall that such a trading strategy consists of buying the call, shorting or selling the stock, lending an amount equal to the sum of the present values of the exercise price and the expected dividends, and holding this portfolio until expiration. Since short selling is not permitted legally in Germany, however, this strategy could have been pursued only by owners of the underlying stock.

Tests concerning the immediate exercise lower bound and the pseudo-American lower bound have not been conducted in Trautmann's study because Geske, Roll and Shastri (1983) demonstrated that this payout protection completely inhibits the early exercise of American call options. Therefore, for the payout protected call options traded in Germany, the pseudo-American lower boundary test parallels exactly the European lower boundary test. On the other hand, violations of the immediate exercise lower bound may not have been observed in Germany for two simple institutional reasons: first, in Germany, a call option cannot be bought and exercised on the same day. Second, the German board-brokers who trade stock options have reportedly prevented call transactions from violating this condition.

3.2 Test of Boundary Conditions Among Different Equity Options and the Underlying Stock

Among all boundary conditions involving more than one option written on the same underlying stock, most interest has been devoted to the relations between the prices of a put and a call option having the same exercise price and expiration date. This relation, termed put-call parity was first proposed and tested by Stoll (1969). By transforming equation (9) into a least-squares regression model, he empirically investigated put-call parity among Over-the-Counter (OTC) options written during 1967 in the U.S. Even though the estimated intercept was significantly higher than the expected value (zero), Stoll (1969, p. 823) reasons that his results are consistent with his model.

Later, Gould and Galai (1974) argued that regression analysis is not an appropriate tool for investigating arbitrage opportunities. Following Merton's (1973b) extension of put-call parity to American options, they claimed that, in the absence of transaction costs, any market violation of relation (10) may indicate market inefficiency. Again, using OTC options data from 1967 to 1969, Gould and Galai observed many violations where the right side of condition (10) exceeded the price of the put. Taking into account transaction costs, these mispricing signals disappeared for nonmembers of the exchange, while members could have made above-normal profits. However, no ex-ante test has been conducted to examine the extent to which the remaining mispricing signals would translate into above-normal profits.

Klemkosky and Resnick (1979) were the first to test the put-call parity relation for American options traded on the CBOE, the American, and the Philadelphia Stock Exchanges. Transactions data for one day each month from July 1977 to June 1978 for fifteen stocks and their dividend unprotected options were used in testing relation (10), modified for dividend payments. Also, the data was screened to eliminate those options which were likely to be prematurely exercised. Violations of the lower and upper boundary (in terms of a call) for a dividend unprotected American put are expressed by the gross terminal profit, ε_4, from engaging in a long hedge position:

$$\varepsilon_4 = (C - P - S)e^{rT} + K + \sum_{j=1}^{n} D_j e^{r(T-t_j)} \qquad (20)$$

and by the gross terminal profit, ε_5, from engaging in a short hedge position:

$$\varepsilon_5 = (S + P - C)e^{rT} - K - \sum_{j=1}^{n} D_j e^{r(T-t_j)} \tag{21}$$

A long hedge position can be established by selling (writing) a call and purchasing an equivalent position obtained by a long position in the underlying stock, buying a put, and financing the purchase in part by borrowing at the riskless interest rate r the amount $Ke^{-rT} + \Sigma_{j=1}^{n} D_j e^{-rt_j}$, where D_j is the dividend payment at date t_j.

In order to approximate the required simultaneity of option and stock prices, Klemkosky and Resnick (1979) considered only those positions whose hedge securities were traded within one minute of each other. From these 606 positions, 66 violated the sufficient condition for no premature exercise of the call option and were eliminated. Of the remaining 540 long hedge positions, 243 guaranteed a non-negative, riskless terminal profit ($\varepsilon_4 > 0$). But, after introducing transaction costs of \$20 per trade for a member of the exchange and \$60 for a non-member, it was found that only 147 member observations and 38 non-member observations of the 540 acceptable observations, respectively, were still profitable.

In perfect capital markets, a short hedge position is obtained by selling a long hedge position (i.e., $\varepsilon_4 = -\varepsilon_5$). The corresponding terminal profit, ε_5, however, equals $-\varepsilon_4$ only if neither the put nor the call option will be prematurely exercised. For a call option, there exists a sufficient condition for not premature exercise, but without sufficient dividends paid exactly at expiration, there is no condition which assures that the put will not be exercised early. Hence, the return on a short hedge where an American put is written is potentially more risky to the hedger than the return on a long hedge where the put is bought. Using the same data set, Klemkosky and Resnick (1979) found that of the original 606 short hedge positions, 240 (127) indicated positive terminal profits, ε_5, in excess of the \$20 (\$60) transaction cost. However, the extent to which rational premature exercise of the put would reduce profits was not examined.

In their 1979 paper, the ex-post analysis was not followed by ex-ante tests. Consequently, the detected "paper" profits may have been solely a result of asynchronous markets. Therefore, in 1980, Klemkosky and Resnick extended the previous work by conducting an ex-ante analysis with the same data set. Hedges constructed five and fifteen minutes after mispricing signals indicated terminal profits in excess of transaction costs of $20 per hedge. Although the long hedge results indicate that only for high ex-post profitability levels the corresponding ex-ante level is significantly lower, the market efficiency hypothesis was not rejected by the authors. This conclusion is not surprising given the results of Phillips-Smith (1980), who showed that even for a member of both the stock and options exchanges, the total level of transaction costs (including opportunity costs of membership on the exchanges and the bid-ask spread differentials) exceeds $70 per contract. Furthermore, for the ex-ante results of the short hedge, profits were significantly lower than the ex-post results on the long hedge investments, but higher than the ex-ante returns. This might be expected, since these higher returns may be regarded as a compensation for the risk associated with the written put in the short hedge position.

Trautmann (1985a) also examines the profitability of long hedge positions consisting of stock and dividend protected options traded on the Frankfurt Stock Exchange and Frankfurt Option Exchange, respectively, during the period April, 1983 to September, 1984. Because of the protection of German stock options against dividend payments, the gross terminal profit from engaging in a long hedge position,

$$\varepsilon_4' = (C - P - S)e^{rT} + K + \sum_{j=1}^{n} D_j(e^{r(T-t_j)} - 1) \,, \qquad (22)$$

is slightly different from equation (20). Unfortunately, the required simultaneity of option and stock prices could not be guaranteed in the German market. Alghough trading in the related assets takes place (almost) simultaneously on the same trading floor, there was no information available about the exact time of each transaction or bid/ask price quotations. For a certain option series at most one transaction price or bid/ask quote is registered per trading day. These prices together with the odd-lot prices (Kassakurse) of the underlying stocks have been used to establish a single long hedge position per trading day and option series.

In the ex-post analysis, 8,714 of 15,507 positions revealed a mispricing magnitude ε_4' , which exceeded the explicit transaction costs of the lowest cost traders. These net terminal profits were used as the signal to trigger the ex-ante establishment of the long hedge.

The thinness of the option market, however, caused serious problems in completing such a position within the next trading day. The long hedge was attempted only when it was possible to buy the put option (the least liquid asset) within the next trading day. Depite this non-simultaneous long hedge construction, there remained some open positions which could not be hedged (even within the next 20 trading days). Nevertheless, 5,802 long hedge positions could be established, and more than 75 % yielded a positive net terminal profit for an exchange member. Major parts of these positive returns, however, may be regarded as a compensation for the risk associated with the nonsimultaneous construction of the ex-ante long hedge position and for the opportunity costs of an exchange membership.

The convexity rule (3), which involves three call options differing only in their exercise prices has been empirically examined by Galai (1979) and Bhattacharya (1983). In the latter study, which takes into account transactions costs, only one violation of this boundary condition was found among the 1,006 triplets of CBOE options observed from August, 1976 to June, 1977. For a comparable sample size, but earlier sample period, Galai (1979) detected twenty-four violations for the zero transaction cost case. The results offer strong support for the market efficiency hypothesis.

3.3 Tests of Equity Option Pricing Models

Option pricing models are constructed in part to explain or predict the absolute price levels of options trading in synchronized, efficient capital markets. The inablility of a model to explain observed market prices may be due to one or more of the following facts:

(1) the mathematical structure of the model is incorrect, or
(2) model parameters have been incorrectly measured, or
(3) the options market is inefficient, or
(4) the markets for the related assets are nonsynchronous.

Although most models fail strict predictability tests, other criteria for judging the model's validity are available. To date, four major approaches for testing an option model's validity have been distinguished:

(1) tests of robustness
(2) tests of predictability
(3) tests of unbiasedness
(4) tests of hedge return behavior

Among these tests, perhaps the most intuitive is to predict market prices with model values using parameters based on historical data. Since casual empiricism indicates that the standard deviation of the stock's rate of return is not constant over time, the historical volatility may not be a good estimate of future volatility. Following an idea of Latané and Rendleman (1976), more sophisticated tests of predictability use the pricing model itself to improve the estimation techniques. By equating the option's actual market price to the model value, and solving (numerically) for the only unobservable parameter, the standard deviation of the stock's return, one obtains the model implied standard deviation (ISD). Provided that (1) the model is correct, (2) option and stock markets are efficient and synchronous, and (3) the ISDs are derived from actual market prices, then the prediction error should approach zero. Nevertheless, sufficient differences between observed prices and model values could exist to permit trading profits. However, a hedging test would have to be conducted to detect systematic market inefficiency since a predictability test only compares accuracy.

Tests of unbiasedness focus on identifying systematic behavior in the prediction error of the valuation models. For some biases (with respect to time and striking price), the concept of ISD has served as the main workhorse, since it helps to isolate the hypothesis of the model's validity. For bias with respect to the volatility, the ISD is not appropriate, as will be emphasized later. Assuming the model to be tested is valid, it is expected that the ISD will be stationary across maturities and striking prices. Deviations from this expectation may be due to the model being misspecified. Furthermore, if volatilities are constant over time, the ISD is also expected to be approximately equal to the historical time-series standard deviation. However, nonstationarity, market inefficiency or lack of synchronous data may cause the deviations from this expectation which are difficult to isolate.

Tests of hedge return behavior empirically examine whether the return on an (almost) riskless hedge position is above-normal. In their pioneering test, Black and Scholes (1972) created a riskless hedge position by buying (selling) one option, and at the same time selling (buying) an appropriate fraction (more precisely, the hedge ratio $N(d_1)$) of the underlying stock. Alternatively, Whaley (1982) formed a hedge position by buying underpriced and selling overpriced options in proportions such that the net investment is zero and no risk is assumed. As long as the hedge return is computed by using market prices, both approaches jointly test the propositions that:

(1) the mathematical structure of the model is correct, and
(2) the model parameters have been correctly measured, and
(3) the options market is efficient, and
(4) the markets for the related assets are sychronous.

Tests of robustness investigate how the results predicted by a model change when its basic assumption are violated or when its input parameters are incorrectly measured. Typically, such a test involves the direct comparison of alternative model values, or the examination of hedge returns calculated (partly or totally) from simulated data. The (not exhaustive) survey of the literature presented in Table 4 indicates that most of the purely empirical studies on option pricing models involve more than a single test. It is, therefore, not possible to unambiguously classify these studies with respect to the testing approach applied. In the following paragraphs, the main results of the empirical testing approaches (1) - (4) are briefly reviewed. Since tests of robustness are useful in interpreting the other test results, these are summarized first.

3.3.1 Results of Robustness Tests

In a recent paper, Ball and Torous (1985a, p. 155) "provide statistical evidence consistent with the existence of lognormally distributed jumps in a majority of the daily returns of a sample of NYSE listed common stocks." However, they find no "operationally significant differences between the Black-Scholes and Merton model prices of the call options written on the sampled stock." Ball and Torous confirm Merton's (1976b) prediction that only for short-maturity and out-of-the-money calls do the jump model prices deviate significantly (up to 100 %). Thus, although most optioned stocks exhibit statistically significant jumps, the jumps are both too small and too frequent to cause option values to differ much from the diffusion model.

Black and Scholes (1972) and Bhattacharya (1980) test whether the deviation of the empirical distribution of stock price changes from the assumed stationary, lognormal distribution affects the expected hedge returns. To isolate this effect, Bhattacharya modifies the empirical hedge return test in two ways. First, as in Black-Scholes original study, instead of an estimate of the stock return volatility, its realization during the option's life was used in the hedge ratio

Table 4

Survey of Empirical Testing Approaches

Empirical and simulative studies on option pricing models (not exhaustive!). The type of testing approach is marked in the following way:

1 = test of robustness, 2 = test of predictability, 3 = test of unbiasedness,

4 = test of hedge return behavior.

Note: OTC = Over-The-Counter Market, SIM = SIMulated Sample Market or period, CBOE = Chicago Board of Options Exchange, AOE = American Options Exchange, FOE = Frankfurt Options Exchange.

Year of Public.	Name of Authors	Sample Market	Sample Period (Year/Month)	Testing Approach			
				1	2	3	4
1972	Black/Scholes	OTC	66/01-69/12	x		x	x
1975	Black	CBOE	73/04-74/12		x	x	
1976	Merton	SIM	SIM	x			
1976	Boyle/Ananthanarayanan	SIM	SIM			x	
1976	Latané/Rendleman	CBOE	73/10-74/06		x	x	x
1977	Galai	CBOE	73/04-73/11				x
1977	Trippi	CBOE	74/08-75/03		x		
1978	Chiras/Manaster	CBOE	73/06-75/04		x	x	x
1978	Finnerty	CBOE	73/04-74/12				x
1978	Schmalensee/Trippi	CBOE	74/04-75/05		x	x	
1979	MacBeth/Merville	CBOE	75/12-76/12		x	x	
1980	Boyle/Emanuel	SIM	SIM	x			
1980	MacBeth/Merville	CBOE	75/12-76/12		x	x	
1980	Bhattacharya	CBOE	76/05-77/10	x			
1982	Whaley	CBOE	75/01-78/03		x	x	x
1982	Gultekin/Rogalski/Tinic	CBOE/AOE	75/01-76/01		x	x	
1982	Emanuel/MacBeth	CBOE	76/01-78/12		x	x	
1983	Blomeyer/Klemkosky	CBOE	77/07-78/06			x	x
1983	Galai	CBOE	73/04-73/11				x
1983	Geske/Roll/Shastri	SIM	SIM			x	
1983	Trautmann	FOE	79/01-83/03		x	x	
1984	Geske/Roll	SIM	SIM			x	
1984	Geske/Roll	CBOE	76/08/24	x	x	x	
1984	Blomeyer/Johnson	CBOE	78/06-78/08		x	x	
1985	Ball/Torous	CBOE	83/01	x			
1985	Rubinstein	CBOE	76/08-78/08			x	
1985	Trautmann	FOE	83/04-85/06		x	x	x

calculation. Second, in the hedge return calculation, the option's market price is replaced by its corresponding model price. This analysis of the hedge returns led Bhattacharya (1980, p. 1,094) to conclude that "if stationarity of the return distributions is assumed and hedge positions are revised daily, then the Black-Scholes formula exhibits no operationally significant mispricing except for at-the-money options with one day to maturity."

While the above-mentioned robustness tests considered empirical stock price movements, in Boyle and Emanuel (1980), stock returns were randomly generated to simulate the assumed lognormal distribution. Their aim is to analyze the deviations from the assumed distribution of continuously hedged returns, when the portfolio is rebalanced at discrete time intervals. They demonstrate that the discrete hedge distribution is particularly skew, and some consequences of this skewness for empirical hedge return tests are explored.

3.3.2 Results of Predictability Tests

Since listed options are American and the Black-Scholes formula is for European options, it is realistic to expect a prediction error attributable to the early exercise component of the price. There are American formulae that can price the early exercise component of option value. These American models which model the early exercise feature as compound options are based on the compound option model of Geske (1979a). Whaley (1982) examines the pricing performance of Roll's (1977) model for American call options on dividend-paying stocks (as modified by Geske (1979b) and Whaley (1981)), and compares it with two suggested approximation methods based on the Black-Scholes model. To analyse each model's power, he uses the model-specific ISDs calculated from the closing stock prices of week $t-1$ and the closing stock prices of week t to predict closing option prices of week t.

For this one-week prediction interval, all three formulas yielded prices which are, on average, within three and a half cents of the observed market price of $4.1388. For each of the 160 weeks in the sample period observed market prices C_j from 91 option classes were regressed on each model's price estimates $\hat{C}_j$, as follows

$$C_j = \alpha_0 + \alpha_1 \hat{C}_j + \varepsilon_j \tag{23}$$

where ε_j is a disturbance term. With perfect prediction, the values of the coefficients α_0 and α_1 in this regression would be indistinguishable form zero and one, respectively. The average values of the weekly parameter estimates $\bar{\hat{\alpha}}_0$ and $\bar{\hat{\alpha}}_1$, ranged from -0.0508 to -0.0300 and 1.0088 to 1.0091, respectively. Whaley (1982, p. 44) concludes that "all of the models seem to perform extremely well, with the explained variation being greater than 98 percent in all cases (i.e., the average of the coefficients of determination from the regressions $\bar{R}^2$ is ≥ 0.98 for each model). However, all models demonstrate a slight tendency to overprice high-priced options (i.e., $\bar{\bar{\alpha}}_1 > 1$).

Although the mispricing errors were small on average, the Black-Scholes model exhibited previously documented systematic biases. The Black-Scholes model tends to undervalue (relative to the market) near maturity call options (i.e., one month to expiration), deep out-of-the-money call options (i.e., $S/K < .75$), and call options on stocks with relatively low historical volatility. The model overvalues call options that are deep in-the-money (i.e., $S/K > 1.25$), and call options on stocks with relatively high historical volatility. Furthermore, MacBeth and Merville (1979) and Rubinstein (1985) have noted that the money bias tends to reverse itself in different time periods, and Geske and Roll (1984a) showed that this reversal of the money bias may be partially attributable to improper treatment of early exercise.

Trautmann (1983b) tested the pricing performance of the Black-Scholes model using dividend-protected FOE options written in the 222-week period before the 1983 changes in trading rules. In this sample period, no organized secondary market for stock options existed. Thus, trading was illiquid, and an option's price could only be consistently observed on the day the contract was written. An observed option price was only considered if within the preceding five trading days at least two contracts have been written on the same underlying stock with the same maturity. All prices observed within this five-day period for options of the same maturity were used to calculate the ISDs are predictors of the future return volatility. The average values of the weekly parameter estimates, $\bar{\bar{\alpha}}_0$ and $\bar{\bar{\alpha}}_1$ ranged from 0.072 to 0.121 and 0.981 to 0.993, respectively, depending on the option's maturity. The average of the coefficients of determination from the cross-sectional regressions $\bar{R}^2$, ranged from 0.915 for 2-month options to 0.953 for 6-month options.

Emanuel and MacBeth (1982) extended the previous research of MacBeth and Merville (1980) in comparing the predictability of the Black-Scholes and CEV model. The root-mean-squared forecast errors indicate that for short-term predictions, the CEV model yields more accurate predictions that the Black-Scholes model. However, "the superiority of the constant elasticity of variance model diminishes as the prediction interval increases. In general, for prediction intervals of one month or longer, the Black-Scholes model does as well as the constant elasticity of variance model" (p. 541). The explanation of this result may be found in the nonstationary behavior of the estimated elasticity parameter ρ in the CEV model.

The American put valuation problem has been addressed by Brennan-Schwartz (1977), Parkinson (1977), Cox, Ross, and Rubinstein (1979), and Geske and Johnson (1984). The empirical work on valuing American puts is less advanced than for American calls, in part because puts werde not listed until June, 1977. Brennan-Schwartz find that putholders do not seem to optimally exercise the put. Farkas and Hoskin (1979), using Parkinson's model on weekly CBOE data from June to December, 1977, find the model to undervalue the puts on average, especially the out-of-the-money options.

In a direct comparison between a European and an American valuation model, using 10,295 CBOE transaction observations from June through August, 1978, Blomeyer and Johnson (1984) establish that the Geske-Johnson American model dominates the Black-Scholes European model. Although the Geske-Johnson American put model will yield the same value as the above alternatives, it is an order of magnitude faster. The fact that the American value is significantly closer to the market price in more than the majority of cases indicates that the early exercise is an important component of the put price.

In summary, it must be concluded that in the case of the most widely traded dividend-unprotected American call options, Roll's (1977) American formula did yield consistently better predictions of market prices than the Black-Scholes European model. Also, the Geske-Johnson American put model appears to dominate the Black-Scholes European model.

3.3.3 Results of Unbiasedness Tests

The Black-Scholes model values do not consistently give unbiased predictions of market prices. The reported biases have occurred with respect to

- the stock's volatility
- the exercise price
- the time until expiration.

Black and Scholes (1972) and MacBeth and Merville (1979) document the phenomenon that options on low-risk stocks are undervalued and options on high-risk stocks are overvalued by the model. Black (1975) reports that in the early years of trading on the CBOE, the model systematically undervalued deep out-of-the money options and near-maturity options while it overvalued deep in-the-money options. However, MacBeth and Merville (1979) and Rubinstein (1985) document a reversal of this striking price bias for the years 1976 and 1977, respectively. That is, the authors conclude that the Black-Scholes model undervalues (overvalues) the prices for in-the-money (out-of-the money) options. This bias acts as if the ISD, the stock volatility implied by an observed option price, were inversely related to the exercise price. But, Emanuel and MacBeth (1982) found that the original striking price bias observed by Black (1975) (which exhibits a direct relation between ISDs and exercise prices) had reestablished itself in late 1977 and 1978.

Geske and Roll (1984a) offered an explanation of these biases based on the dividend-induced early exercise feature of American call options. In fact, Whaley (1982) does demonstrate empirically that these biases are reduced when Roll's (1977) American call option formula is employed. For each model Whaley tested, the following cross-sectional regressions were estimated for each week during the 160-week sample period to regress the relative prediction error on the stock's volatility, the extent to which the option is in- or out-of-the-money, and the time until expiration:

$$\frac{C_j - \hat{C}_j}{\hat{C}_j} = \alpha_0 + \alpha_1 \hat{\sigma}_j + \varepsilon_j \tag{24}$$

$$\frac{C_j - \hat{C}_j}{\hat{C}_j} = \alpha_0 + \alpha_1 \left(\frac{S_j - K_j e^{-r_j T_j}}{K_j e^{-r_j T_j}}\right) + \varepsilon_j \tag{25}$$

$$\frac{C_j - \hat{C}_j}{\hat{C}_j} = \alpha_0 + \alpha_1 T_j + \varepsilon_j \quad . \tag{26}$$

Roll's (1977) corrected formula serves to reduce the magnitude of the slope coefficient α_1 and the coefficient of determination in the cross-sectional regressions (24), (25), and (26), but the hypothesis that there is no relationship between the relative prediction error $(C_j - \hat{C}_j)/\hat{C}_j$ and the volatility estimate $\hat{\sigma}_j$ is soundly rejected. Contrary to previous evidence, for each of the three models compared, the extent to which the option is in-the-money or out-of-the-money apparently does not significantly affect the model's prediction error. The results of regression (26) show that while there exists a significantly negative relationship for the valuation method based on the Black-Scholes formula (i.e., it undervalues near maturity options and overvalues longer maturity options), it virtually disappears when Roll's model is used.

Geske and Roll (1984a) suggest that dividend uncertainty may explain a small component of the variance bias exhibited by both the Black-Scholes European and Roll's American model. Trautmann (1983b), however, demonstrates empirically with FOE data that the consideration of this aspect does not remove this bias. By constructing a subsample of 19,040 call options with no dividend payments expected prior to expiration, it was found that in this subsample the variance bias is even more significant than in the total sample comprising 28,299 options.

Rubinstein (1985) examines whether one of the alternatives to the Black-Scholes model listed in Table 1 can explain the observed striking price and time-to-maturity biases of the Black-Scholes values. The null hypothesis is that the Black-Scholes formula produces unbiased values and hence ISDs should be constant for options on the same underlying stock with different maturities and striking prices. By comparing simulated and observed ISDs across maturities and striking prices, he tries to distinguish which pricing formula seems to better explain the observed biases from Black-Scholes values. This methodology avoids the difficult task of estimating the stock volatility. Furthermore, to circumvent possible objections of regression analysis with data that is not normally distributed, nonparametric tests were conducted since they require no assumptions about the population from which the observed sample is drawn. According to this test design, Rubinstein argues that the main measurement problem is the simultaneity of the market prices for the option and the underlying stock. To surmount these problems, the data are taken from Berkeley Options Transactions Data Base. This is a time-stamped record (to the nearest second) of all reported trades, quotes, and volume on the CBOE during the day.

Examination of the observed ISDs from August 23, 1976 to August 31, 1978, confirms the previous finding that, for an out-of-the-money call, the shorter the time to expiration, the higher its ISD which means it was relatively overpriced. This is the only result which is consistent over the whole sample period. The other conclusions depend on the subperiod considered (first subperiod: August 23, 1976 to October 21, 1977, second subperiod: October 24, 1977 to August 31, 1978), and reveal reversals in both the time and money bias observed in each subperiod.

These conclusions confirm the previous findings of MacBeth and Merville (1979) that the direction of the striking price bias changes from time to time. More importantly, Rubinstein concludes that no one alternative model seems to remove all the observed Black-Scholes biases. His proposal is to build a composite model which might, in addition, depend on some macroeconomic variables, such as the level of stock market prices, the level of stock market volatility and the level of interest rates.

Geske and Roll (1984b) repeat Whaley's tests using the Berkeley Options Transaction Data for all options traded at midday on August 24, 1976, resulting in a sample of 667 different options on 85 stocks. Options on the same stock differed by exercise price, expiration dates, and scheduled dividend payments prior to expiration. A subsample of 119 options in 28 different stocks with zero scheduled dividends during their remaining life was identified within the main sample. Using regression analysis, Geske and Roll demonstrate the original time, money, and volatility biases are present in the entire sample. Next, they show that in the nondividend subsample, the time and money biases are significantly reduced, but the volatility bias remains large. However, by correcting the volatility estimates of all stocks for errors in variables by "shrinking" the volatility of each stock in their sample toward the mean of the sample in a manner suggested by Stein (see Effron and Morris, 1975) the volatility bias is reduced. Thus, Geske and Roll conclude that the time and money biases may be more related to improper model treatment of early exercise while the volatility bias may be more related to estimation problems than to model assumptions (see Section 3.5).

3.3.4 Results of Hedge Return Behavior Tests

Most of the studies using option valuation models to identify overvalued/undervalued call options and then testing for market efficiency are based on the hedging technique of Black and Scholes (1972). That is, if the model value C^M is higher (lower) than the market price C, the call is underpriced (overpriced), and a long (short) position is taken in the call with a corresponding short (long) position is in the stock. If, in an ex-post test, the hedge position is established at time t and liquidated at $t+1$, the ex-post hedge return is

$$R_{H,t+1} = \begin{cases} (C_{t+1} - C_t) - N(d_{1,t})(S_{t+1} - S_t) & \text{if } C_t < C_t^M \\ N(d_{1,t})(S_{t+1} - S_t) - (C_{t+1} - C_t) & \text{if } C_t > C_t^M \end{cases} \tag{27}$$

The hedge return minus the opportunity cost $(e^{r\Delta t}-1)I_{H,t}$ on the investment

$$I_{H,t} = \begin{cases} C_t - N(d_{1,t})S_t & \text{if } C_t < C_t^M \\ N(d_{1,t})S_t - C_t & \text{if } C_t > C_t^M \end{cases} \tag{28}$$

will give the excess return for the hedged position.

Working with OTC data, Black and Scholes (1972) adjusted the hedge position daily by buying or selling shares of the underlying stock, assuming no transaction costs. Since a secondary market in OTC options was virtually nonexistent, they use their model to simulate the market prices needed for the calculation of daily hedge returns. When regressing the observed positive excess returns against a market index, no significant systematic risk was detected. When transactions costs were included, the substantial positive excess returns detected in the zero transaction cost case vanished.

Galai (1977) repeated this ex-post test with CBOE options by adjusting the options' positions. The hedge return averaged over all option contracts considered was \$9.80 per option contract (on 100 shares) per day. Furthermore, he performed an ex-ante test where, on day $t-1$, it is decided whether the option was over or underpriced, and the hedge ratio was calculated. The hedge position, however, is established and liquidated on day t and $t+1$, respectively. As expected, the average of the hedge returns fell from \$9.80 to \$5.00 per option contract per day for the ex-ante test. While in the ex-post test for 71 option series out of 202, the average of the time-series hedge return was significantly different from zero (at the 5 percent level of significance), this number dropped from 71 to 12 in the ex-ante test. Since the average opportunity costs for the hedge investment (about \$0.30 per contract per day) were found to be negligible, only the inclusion of transaction costs might eliminate the remaining positive hedge returns.

Similar magnitudes of excess hedge returns were observed when a spreading strategy was simulated. A spreading strategy consists of a long position in an undervalued option on the same underlying stock. Galai (1977, p. 195) concludes that "the market did not seem perfectly efficient to market makers" but,... "it does not seem that a non-member of the CBOE can expect to achieve above-normal profits consistently."

Chiras and Manaster (1978), in a similar study, use ISDs instead of historical volatilities to identify over or underpriced CBOE options with respect to the Black-Scholes values. A spread position is established for those options whose theoretical values deviate by at least 10 percent from the market price. The amount of each option included in the hedged position is determined by its hedge ratio and all established positions are liquidated one month later. This spreading strategy is used to eliminate the effect of stock price movements on the hedge returns. Unfortunately, the requisite of being able to identify both an underpriced and an overpriced option on the same stock restricts the number of options that may be included in the sample. Although they are careful to realize the ex-post nature of their tests, the positive excess hedge returns led them to conclude that even non-members of the CBOE could have made above-normal profits in the sample period. Using the data of Chiras and Manaster, Bookstaber (1981) and Phillips and Smith (1980) demonstrate, however, that the paper profits observed by Chiras and Manaster disappear when the nonsimultaneity of option prices and the bid-ask spread is considered, respectively.

In Whaley (1982), the validity of Roll's (1977) American call option model is tested jointly with the propositions that the previous week's ISD is an accurate reflection of the expected volatility and that the related markets are efficient. Roll's model is used to identify underpriced/overpriced options, and then a hedge portfolio is created by buying underpriced and selling overpriced CBOE options, such that the portfolio's net investment and its systematic risk is zero. Whaley rebalances the portfolio weekly, and thus the sample includes 160 weekly hedge returns. At the 0.0115 percent significance level, the null hypothesis of a zero hedge return was rejected. However, a proportional transaction cost rate of 0.616 percent was sufficient to eliminate all the profits that could be realized by following the cost-less trading strategy. Since the Phillips and Smith (1980) estimate of the bid-ask spread transaction cost component exceeds this critical rate, option market efficiency is soundly supported.

Blomeyer and Klemkosky (1983) also compare the ability of the Black-Scholes and Roll option pricing models to identify overpriced and underpriced call option contracts by using the hedging technique suggested by Black and Scholes (1972). The option positions in the hedge portfolio are adjusted for each new option transaction price, and the returns are calculated over the intervals between successive transactions. Since the Roll model allows for the early exercise of CBOE options, it is expected that the Roll model should outperform the Black-Scholes model for options written on high-dividend-yield stocks. However, the model-specific hedge results do not differ significantly. For both models, the ex-post excess hedge returns are significantly positive over most trading days and underlying securities. Even the ex-ante analysis produced significant positive returns on a before-transaction cost basis. But, these average profits disappear after the returns are adjusted for transaction costs.

3.4 Tests of New Option Instruments

In the 1980s, the Securities and Exchange Commission (SEC) granted permission to begin trading put and call options on foreign currency exchange rates, on futures on stock indices, currencies, and commodities, and on short-term Treasury Bills, and long-term Treasury Bonds. The volume in these markets is expected to increase as more investors become aware of their potential. The previously discussed theory of pricing options on equities applies to these new option instruments, with some changes.

For example, call option on currencies and futures have a positive probability of early exercise. The intuition regarding early exercise of American calls on currencies and futures can be attributed to an implied dividend effect with reasoning similar to the equity case. The implied dividend for currencies is the differential risk-free interest rate in the foreign and domestic countries, and is the cost of carry in the futures market. The debt options are more difficult to value than equity options because of the fixed maturity and fixed payout, and because it is not sensible to assume constant interest rates. There are several empirical papers testing valuation models for the new options.

For the currency options, Bodurtha and Courtadon (1984) test the efficiency of the Philadelphia Stock Exchange Foreign Currency Options Market using transactions data during the time period from February, 1983 to September, 1984. They find violations of the early exercise boundaries and of the put-call parity condition. However, when bid-ask spread transaction costs and simultaneous prices are used, most of the violations disappear. Shastri and Tandon (1984a) test a version of the Geske-Johnson American Options model applied to currency options for both puts and calls using the same Philadelphia data. They find systematic deviations between market prices and model values with respect to the time to expiration and the amount in or out-of-the-money. The model tends to undervalue out-of-the-money options, long maturity puts and short maturity calls, while it overvalues in-the-money options, short maturity puts, and long maturity calls. A hedging strategy returns abnormal profits, but these are eliminated when the bid-ask spread is properly considered.

Whaley (1984b) examines the prices of options on futures using transactions data from the Chicago Mercantile Exchange (CME) on the S & P 500 futures options from January to December, 1983. He compares the Black European futures option model to a version of the Geske-Johnson American model using about 15,000 observations for calls and 14,000 for puts. Whaley shows that both models undervalue in-the-money options and options with a long time to maturity. He demonstrates that the European model's bias is larger and more statistically significant than the American model. Finally, he shows that the market is efficient for retail customers who pay the bid-ask spread. Shastri and Tandon (1984b) also examine options transactions data from the CME on S & P 500 and Deutsche Mark futures during the time period of February, 1983 to December, 1984. Like Whaley, they use a version of the Geske-Johnson American option model adjusted for futures options. They demonstrate that biases exist with respect to the exercise price and time to expiration. They also confirm market efficiency by showing that hedging profits vanish when the bid-ask spread is considered.

Dietrich-Campbell and Schwartz (1984) test the Brennan/Schwartz debt option model on Treasury bill and bond option data collected from closing prices reported in The Wall Street Journal, for the year from November, 1982 through October, 1983. They report that the Brennan/Schwartz model overvalues put and call options on long maturity bonds. The calls and overvalued by 33 cents and the puts by 30 cents, on average. This is better than an application of Black-Scholes, which is shown to overvalue the same options by 57 and 55 cents, respectively. For Treasury Bill options, the Brennan/Schwartz model misvalues the puts and calls by 4.3 and 3.1 cents, respectively. The errors are catalogued with respect to striking price and time to maturity. (It would be useful to know the percent error, but this work is preliminary.) Their paper also supports efficiency in the debt options market by showing that profits disappear when bid-ask spreads are considered.

3.5 Estimation Problems

A theoretical option value is a function of several variables which must be observed or estimated. As we have seen, this includes the price of the underlying asset, the exercise price, time to expiration of the option, the interest rate over the life of the option, the volatility of the underlying asset, and any payouts to the underlying assetholder scheduled during the option life. Typically, the exercise price and the option expiration date are contracted and known with certainty, and have no estimation problems.

The price of the underlying asset can be observed, albeit imperfectly, if it is trading in a market. This measurement error regarding the exact location of the underlying asset price (at or within the bid-ask spread) is further complicated by the lack of simultaneous trading in the option and the stock markets (or currency, futures, or bond markets). Transactions data, time stamped in both markets, with recorded trades indicating at the bid or ask, are the best data financial economists can get to alleviate this measurement problem. The interest rate is generally assumed constant over the option life. For short-lived options (less than one year) this assumption may be the best way to treat an otherwise difficult problem. Which interest rate is used in the hedge process, and differential borrowing and lending rates further complicate this estimation problem. However, estimation of the interest rate does not appear to be a serious problem for equity options since their values are not very sensitive to changes in interest rates (debt options are obviously an exception). Also, scheduled dividend payments over the life of the option are uncertain, but for short-lived options, this uncertainty is small. The option value is most sensitive to the volatility of the underlying asset. Since volatility is not directly observable, it must be measured. Measurement of volatility is probably the most important estimation problem in valuing options, and it will be the focus of this brief subsection.

It is well-known that an unbiased estimate of the volatility will produce a biased estimate of an option value (Thorpe, 1976). The bias in the option value arises because the option formula (Black-Scholes, et al.) are nonlinear functions of the volatility, and unbiasedness is not preserved under nonlinear transformations. However, this source of bias in the option price is not large, as Boyle/Ananthanarayanan (1977) document. Furthermore, Butler/Schachter (1985) demonstrate how this transformation bias can be eliminated.

A more serious potential problem in variance estimation is errors in variables. It is well-known that the Black-Scholes model undervalues options on low variance stocks and overvalues options on high variance stocks. Geske and Roll (1984b) point out that the variance-related mispricing always arises in the context of an inter-stock comparison, in contrast to the striking price and time-to-exiration biases, which are detected in an inter-option comparison. Unlike the striking price and time until expiration, the true variance is identical for all options on the same stock on a given date. Thus, investigations of variance-related mispricing cannot rely on the implied variance (Latané and Rendleman, 1976), but must instead be based on historical estimates of actual stock return volatility.

There are many techniques to improve the volatility estimate for a single stock. Parkinson (1980), Garman and Klass (1980), and Beckers (1983), use more information (high, low, open, and close for the day) in the estimate. But, the essence of the present problem is that a number of variances are estimated simultaneously, one for each stock, and then option mispricing is related cross-sectionally to these several estimates. Geske-Roll (1984b) demonstrate that by optimally "shrinking" each individual stock variance estimate to the grand mean of all estimates, the observed volatility bias is eliminated.

4. Appendix: Formulae for the Evaluation of European Calls

In this Appendix, formulae are given for the evaluation of a variety of European option pricing models subject to the motion equations listed in Table 1. For a more detailed explanation of each model the reader is referred to the original paper mentioned in Sec. 2.2.3. We start with model number 2 of Table 1, the binomial model, since the Black-Scholes diffusion model was already given in Sec. 2.2.2.

Binomial Model

According as the unit of time is chosen there are two different ways to present the model and the resulting valuation formula.

- Alternative 1: Unit of time is the interval between two successive jumps (for instance one day). The time T to the expiration of the call is an integer number of such units of time. Of course, the risk-free rate of interest r has to refer to this small unit too.
- Alternative 2: Unit of time is - as usual - one year. If the number of jumps occurring up to the expiration of the call is denoted by n, an evaluation formula results which depends on the number n fixed beforehand.

Alternative 2 is more appropriate to analyze the limiting behavior $(n \to \infty)$ provided that the jump size is suitably related to n, whereas alternative 1 is the natural extension of the simple two-state one-period model to the T period case. The following description is based on alternative 1.

$$C = S \cdot B(a;T,p') - K \cdot R^{-T} \cdot B(a;T,p) = \text{call value} ,$$

where

S = stock price at current date

K = exercise price

T = time to expiration

$R = 1 + r$ = one plus the risk-free rate of interest

$p = \frac{R-d}{u-d}$ (sometimes denoted as 'hedging probability')

$p' = \frac{u}{R} p$

u = multiplicative factor of up jump (i.e. the jump is from S to $u \cdot S$) ; $u > R$

d = multiplicative factor of down jump; $d < R$

$$B(a;T,p) = \sum_{j=a}^{T} \binom{T}{j} p^j (1-p)^{T-j}$$ = complementary binomial distribution function

a = smallest non negative integer greater than

$$\ln\left(\frac{K}{Sd^T}\right) / \ln\left(\frac{u}{d}\right),$$

if a happens to be greater than T, the call value C turns out to be zero.

Constant Elasticity of Variance (CEV) Model

$$C = S \cdot \sum_{n=1}^{\infty} g(n,x)G(n+\lambda,y) - K \cdot e^{-rT} \sum_{n=1}^{\infty} g(n+\lambda,x)G(n,y),$$

where the (new) symbols have the meaning:

$$\lambda = \frac{1}{2(1-\rho)}$$ ($\rho \neq 1$ is the exponent of the motion equation in Tab. 1)

$$x = \frac{2\lambda r}{v^2(e^{rT/\lambda} - 1)} S^{1/\lambda} e^{rT/\lambda}$$ (v is the coefficient of the motion equation in Tab. 1)

$$y = \frac{2\lambda r}{v^2(e^{rT/\lambda} - 1)} K^{1/\lambda}$$

and

$$g(n,z) = \frac{e^{-z} z^{n-1}}{\Gamma(n)}$$ = gamma density function

$$G(n,w) = \int_w^{\infty} g(n,z)dz$$ = complementary standard gamma density function.

Compound Option Model

$$C = V \cdot N_2(h,k;\rho) - M \cdot e^{-rT_M} N_2(h-v'\sqrt{T},k-v'\sqrt{T_M};\rho) - K \cdot e^{-rT} N(h),$$

where

V = current market value of the firm

M = maturity value of the firm's zero coupon debt

T = time to expiration of the option

T_M = time to maturity of the zero coupon debt (where $T < T_M$)

$\rho = \sqrt{T/T_M}$

and

$$h = \left[\ln\left(\frac{V}{\bar{V}}\right) + rT + \frac{1}{2}v'^2T\right]\frac{1}{v'\sqrt{T}}$$

$$k = \left[\ln\left(\frac{V}{M}\right) + rT_M + \frac{1}{2}v'^2T_M\right]\frac{1}{v'\sqrt{T_M}}$$

v' = standard deviation of rate of return on the firm's asset

$\bar{V}$ is the value of V defined by the equation:

$$V\cdot N(k) - M\cdot e^{r\tau} N(k-v'\sqrt{\tau}) - K = 0 \quad (\text{where } \tau = T_M - T) .$$

Finally,

$$N_2(h,k;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{h}\int_{-\infty}^{k} \exp\left\{-\frac{1}{2}\,\frac{x^2 - 2\rho xy + y^2}{1-\rho^2}\right\} dy\,dx$$

= bivariate normal distribution function with mean vector = (0,0) and variances = 1 .

$$N(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{k} e^{-\frac{x^2}{2}} dx$$ = univariate standard normal distribution function.

Displaced Diffusion Model

(Special Case: No dividend payments)

$$C = a\cdot S\cdot N(x) - (K - bS)e^{-rT} N(x-v'\sqrt{T}) = \text{call value} ,$$

where

$$x = \frac{\ln\left(\frac{aS\,e^{rT}}{K-bS}\right)}{v'\sqrt{T}} + \frac{1}{2}v'\sqrt{T}$$

$a = \alpha(1+\beta)$

$b = (1-\alpha-\alpha\beta)e^{-rT}$

v' = standard deviation of rate of return on the firm's risky asset

α = fraction of the total firm value currently invested in the risky asset; accordingly, $1-\alpha$ is the fraction invested in the riskless asset.

β = firm's debt-equity ratio.

Jump Process Model

$$C = S\cdot P(x;\lambda) - K\cdot e^{-rT} P(x;\lambda/k) = \text{call value},$$

where

λ = expected jump frequency per unit time

k = expected jump amplitude

x = nonnegative integer representing the smallest number of Poisson stock price jumps, such that the option is in-the-money at the expiration date (i.e. $S(T) > K$)

$P(x;\lambda) = \sum_{i=x}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!}$ = complementary Poisson distribution function with argument x and parameter λ.

Diffusion-Jump Model

(Special case: The jump amplitude has a lognormal distribution) Assuming the jumps have no market component and are thus completely diversifiable, then

$$C = \sum_{i=o}^{\infty} \frac{e^{-\lambda' T} (\lambda' T)^i}{i!} C_i(S,K,T,\sigma_i,r_i) = \text{call value},$$

where

C_i = Black-Scholes value of a European call with time to expiration T and striking price K on a stock with current price S and volatility σ_i, conditional on knowing that exactly i Poisson jumps occur during the life of the option.

$\sigma_i = \sqrt{v^2 + \delta^2 \frac{i}{T}}$

δ^2 = variance of the jump component

$\lambda' = \lambda(1+k)$

λ = expected jump frequency per unit time

k = expected jump amplitude

$r_i = r - \lambda k + \frac{i \, \ell n \, (1+k)}{T}$.

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