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OPTRAD: A Decision Support System for Portfolio Management in Stock and Options Markets

S. Trautmann
Institut für Entscheidungstheorie und Unternehmensforschung, Universität Karlsruhe
Postfach 6980, 7500 Karlsruhe

Summary

OPTRAD (OPtion TRading ADvisor) is a prototype of a decision support system which helps to deal successfully in stock options and their underlying stocks. OPTRAD is especially designed for all kinds of market participants—for private investors as well as banks and other option dealers—acting at the Frankfurt Securities Exchange. At the present stage of completion OPTRAD integrates the following functions:

1. Automatic market data recording (almost in real-time) via BTX (BildschirmTeXt) communication.
2. Option valuation according to modern option valuation theory.
3. Buy/sell recommendations under consideration of institutional and financial restrictions.
4. Positionkeeping.
5. Daily and monthly portfolio revaluation.

The main program of OPTRAD is implemented in the declarative programming language PROLOG while some subroutines are written in the procedural language PASCAL. OPTRAD is running on IBM compatible Personal Computers (PCs) and needs at least 512 KB random access memory.

1 Introduction

In the last few years we have observed a phenomenal development of the options markets, mainly in the U.S. but also in other industrialized countries like Germany. An option is a contract conveying the right, but not the obligation, to buy (call option) or sell (put option) a specified financial instrument (the underlying asset) at a fixed price (exercise or strike price) before or at a certain future date (expiration or maturity date). There are two parties to an option contract: the option seller (writer or grantor) and the option purchaser (buyer or holder). The buyer purchases from the writer a commitment that the option writer will stand ready to sell or purchase a specified amount of the underlying instrument on demand. The price of this right which the option purchaser has to pay to the option seller is called option price.

Options are purchased and traded either on an organised exchange or in the over-the-counter (OTC) market. Exchange-traded options are standardized contracts on specified underlying instruments, like individual stocks, stock indices, precious metals, foreign currencies, bonds, treasury bills, futures contracts, with standardized amounts, exercise prices and expiration dates. OTC option specifications are generally negotiated as to the underlying instrument, amount, exercise price and exercise date. The rapid growth of both types of options markets indicates that many investors have found that at least one of the following reasons applies to them:

1. Options may offer a pattern of returns that could not be obtained with the underlying asset.

2. Options may provide a means of portfolio insurance.

3. Options may allow lower transactions costs than the underlying asset.

4. Options may offer tax advantages unavailable with the underlying asset.

Parallel to this rapid development of options markets there has been a corresponding progress in academic research concerned with these new securities. In two seminal papers published in 1973, Black and Scholes (1973) and Merton (1973a) provided the basic framework of modern option pricing theory. The option pricing models based on it are constructed in part to explain or predict the absolute price levels of options traded in synchronized and efficient capital markets. The observed precision of these valuation models relies primarily on preference-free, enforcable arbitrage conditions. Especially the Black/Scholes formula for valuing European-type stock options has become a widely used formula by practitioners in the options industry to guide their trading decisions: underpriced options (that is, the market price is below the corresponding model value) are considered to be good buys, and overpriced options (that is, the market price is above the corresponding model value) are considered to be good sells.

In this paper, we present a comprehensive option management tool called OPTRAD (OPtion TRading ADvisor) which helps to detect mispriced options according to modern option pricing theory. Furthermore, OPTRAD is designed to support the professional trader, broker, hedger, and portfolio manager as well as the private investor in dealing, investing, risk reducing, profit protecting and hedging strategies involving the use of options. The OPTRAD version described below has been made for users dealing in stock options and their underlying stocks at the Frankfurt Securities Exchange (Frankfurter Wertpapierbörse). The potential benefits of this OPTRAD version include the following:

1. Automatic market data recording via BTX communication. This enables the user of OPTRAD to analyse the mass of daily price data very quickly and at low cost in comparison to other real-time information services.

2. Calculation of fair values for stock options according to modern option pricing theory. Stock options which are seemingly mispriced with respect to their fair values are selected as candidates for purchase or sell recommendations.

3. OPTRAD recommends those stock options and stocks which should be bought or sold, under consideration of institutional and financial restrictions, and dependent on the user's risk preference.

4. Positionkeeping and portfolio revaluation at arbitrary instants provide solid support for trading, control, accounting and portfolio performance measurement.

The following sections describe OPTRAD's present stage of completion, outlining some of its major components and the programming environment. The main program of OPTRAD is implemented in the declarative programming language PROLOG while some subroutines are written in the procedural language PASCAL. OPTRAD is running on IBM compatible Personal Computers (PCs) and needs at least 512 KB random access memory.

2 Automatic Market Data Recording Via BTX (BildschirmTeXt) Communication

The fast detection of mispriced stock options requires the use of a real-time information service to get the actual market data. The German BTX (BildschirmTeXt) system, for instance, delivers the market data quoted at the Frankfurt Securities Exchange in almost real-time and at low costs compared with other information services like Reuters and so on.

BTX is a service offered by the German Federal postal offices (Deutsche Bundespost) since September 1983. BTX informations are transmitted through the public telephone net where the user can call information from the system and can send information to the system, too. The information and communication technology BTX is offered in many other countries under the name Videotex and is based on the British Viewdata System (see Deutsche Bundespost (1984)).

A BTX box and a BTX decoder is needed for a connection with the BTX net. These devices enable the BTX user to communicate with the central mainframe computer located at Ulm. In this machine all BTX information is organized in pages, which can be called by the user. Dependent on the type of BTX terminal the dialog occurs via a small keyboard (e.g., the remote control of a TV set) or an intelligent terminal (e.g., a homecomputer, personalcomputer or BTX terminal). Furthermore, it is possible to connect a mini or mainframe computer with the BTX system. Such a computer is able to communicate with many BTX users via Datex-P at the same time.

In this application the BTX terminal has to be an IBM compatible PC or AT with corresponding BTX hard- and software. This PC is used to receive the current stock and stock option prices via BTX and to save them on the internal fixed disk drive of the PC. It makes no difference which BTX decoder is used in the PC. The software of this application works with all decoders. During the development of this application we used the VC 2000 decoder from IMR (1987), but the installation of a different decoder is easy. A list of all certified decoders can be found under BTX page *1043031#.

In the following we will look at the information fed into the BTX system by the "Frankfurter Wertpapierbörse". The information offered by the "Frankfurter Wertpapierbörse" includes beside many other market price data the daily stock and stock option prices quoted at the Frankfurt Securities Exchange. The available market price information is structured as shown in figure 1. The opening and closing stock prices and all transaction prices between them (Fortlaufende Notierungen) you can get under

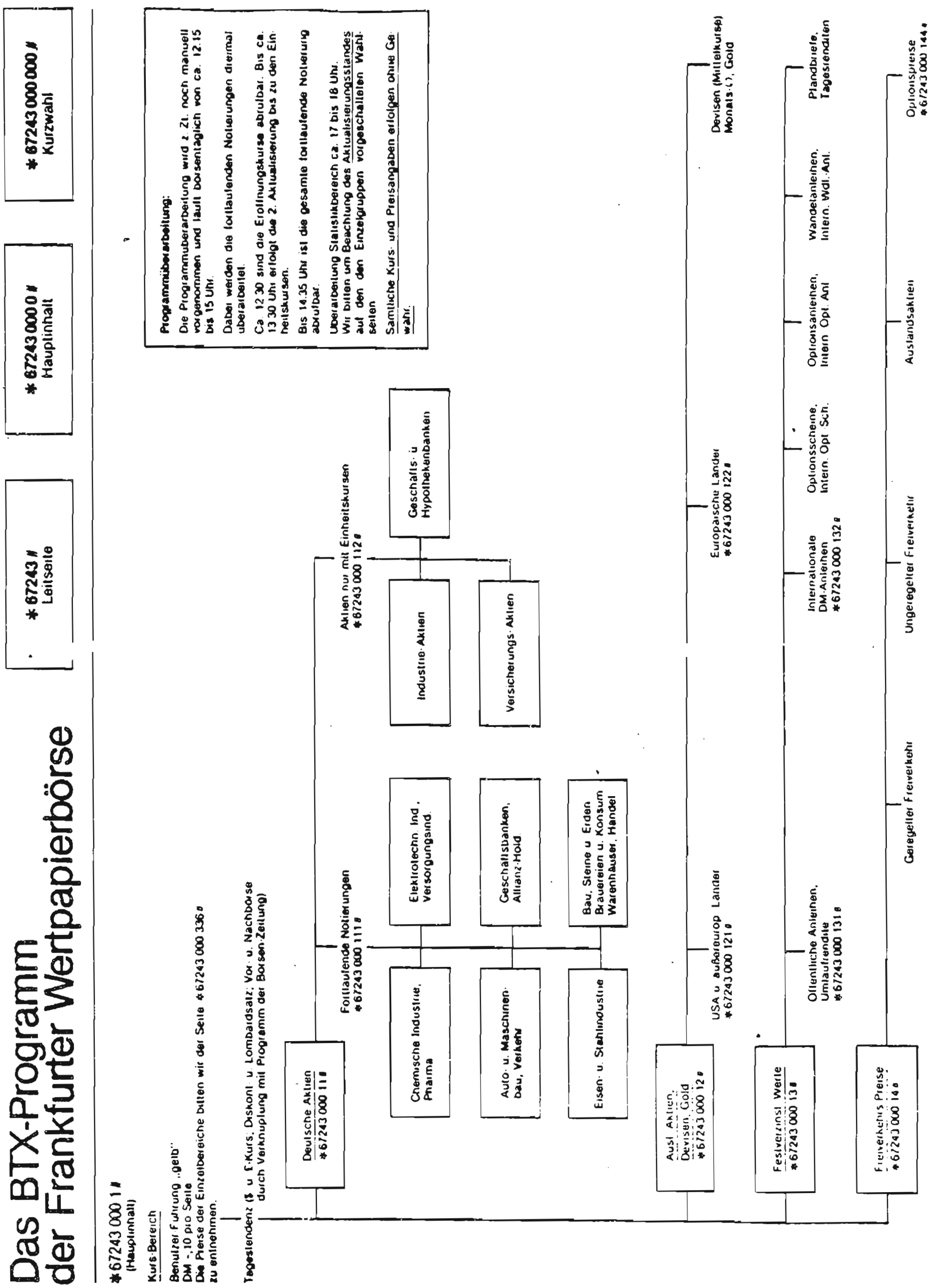

Figure 1: Structure of the market price information offered by the Frankfurter Wertpapierbörse via BTX.

page *67243000111#. These pages contain also the odd lot prices (Kassakurse). In the bottom right of figure 1 you can find the number *67243000144# representing the page with the corresponding stock option prices. Every trading day from half past eleven a.m. until four o'clock p.m. all values are updated. At the moment these data are entered manually in the BTX system, but it is already announced to be done automatically by a computer. All other market data needed by OPTRAD are available through BTX, too. This includes date and amount of dividend payments, subsidiary rights, interest rates for riskless borrowing and lending and so on. These data are available under BTX page *20027# from the "Börsen-Zeitung".

Figure 2 shows the first three BTX source data pages from June 24, 1987 for the chemistry branch stock option prices (BTX page *672430001441#). The first section, for instance, beginning with the symbol BAS comprehends all quoted prices for options written on BASF stocks. Call prices (following a "Kx:") are listed with decreasing maturities "x" while the put prices (following a "Vx:") are listed with increasing maturities "x". The first exercise price/option price entry in figure 2 ("360/4") means that at least one call option contract with exercise price K=DM 360 and expiration date January 15, 1988 was transacted at a price of DM 4. Price quotations without an additional mark denote transaction prices. Additional marks denote either bid or ask prices and are described under BTX page *672430003#.

```
Frankfurter Wertpapierboerse       0,50 DM


 Optionspreise
 Bereich: Chemie

 BAS K9:360/4 340/8TA 320/15TA 300/21TA
280/30 260/43 K6:360/3 340/5,4 320/10TA
300/16TZ 280/32TA 260/46G 240/64G K3:320
/2,7TA 300/6TZ 280/22TZ 260/41,5TZ 240/6
1,5TZ v3:320/18G 300/2TZ V6:320/20G 300/
9TZ 280/2,5G V9:320/27G 300/10TZ 280/7G


 BAY K9:400/5 380/7,5 360/10 340/20 320/
32TA 300/40G K6:400/2B 380/4 360/5,1TZ 3
40/15 320/26 300/42TZ 280/60TZ k3:360/2T
A 340/4,1 320/15TZ 300/36TZ 280/56TZ 260
/76G v3:340/7TZ V6:340/13G 320/6 V9:360/
35G 340/20 320/8,1TZ


 0 Zurueck                    weiter _
                          672430001441a

Frankfurter Wertpapierboerse       0,00 DM
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HFA K9:340/10 320/15 280/31TZ 260/50G K
6:340/5 320/10 310/15 290/19 280/25,1TZ
270/37TZ 270/35 260/45G 250/51TZ K3:320/
2,5TA 310/3,6 300/6,9TA 290/14TZ 280/25T
Z 270/33TZ 250/52 230/72 V3:300/7TZ V6:3
00/16TZ 280/8B 270/5,4B V9:300/18B 280/9
B

SCH K9:600/23 K6:550/31

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                             672430001441b

Frankfurter Wertpapierboerse      0,00 DM

 DGS K9:650/5G 550/17B 600/13B K6:500/2
0,5 K3:490/4B

 SDF

 HEN
 RUE
 WAD3

 FDN K9:320/13TA 300/22G K6:320/10TA K3:
320/5B 30/77TA 280/28B

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                             672430001441c
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Figure 2: The first three BTX source data pages from June 24, 1987 for the chemistry branch stock option prices (BTX page *672430001441#).

Receiving the BTX data is done by calling the batch file HOLDATEN. This file is first calling the option data and thereafter the stock prices. If a different decoder than the IMR VC 2000 is used, this batch file must be modified. The BTX data files are not saved as normal ASCII (American Standard Code for Information Interchange) code, they are saved as ISO code (International Standardization Organisation). Therefore the first step after receiving the BTX data is the conversion from ISO to ASCII code: This conversion is done by the programm AUTOBTX. The result of this conversion is saved in the files AKTIEN.TXT and OPTIONEN.TXT. Since the main program of OPTRAD is written in PROLOG, the BTX ASCII-data files have to be converted into files of PROLOG facts. This is done by two conversion programs called WANDEL-A and WANDEL-O: WANDEL-A converts the stock data, while WANDEL-O converts the option data. Figure 3 shows the converted data, that means the PROLOG data file, which has been built through the conversion of OPTIONEN.TXT. "opxt" is a relation containing all call (x = k) or put (x = v) option data with maturity t written on a certain stock. opk9("bas",[...]), for instance, contains the exercise price and option price with corresponding price type code (e.g., price type code "0" denotes a transaction price) for all call options with maturity 9 (expiration date January 15, 1988) written on BASF stocks.

```
opk9("bas",[360,4,0,340,8,1,320,15,1,300,21,1,280,30,0,260,43,0])
opk6("bas",[360,3,0,340,5.4,0,320,10,1,300,16,2,280,32,1,260,46,
3,240,64,3])
opk3("bas",[320,2.7,1,300,6,2,280,22,2,260,41.5,2,240,61.5,2])
opv3("bas",[320,18,3,300,2,2])
opv6("bas",[320,20,3,300,9,2,280,2.5,3])
opv9("bas",[320,27,3,300,10,2,280,7,3])
opk9("bay",[400,5,0,380,7.5,0,360,10,0,340,20,0,320,32,1,300,40,3])
opk6("bay",[400,2,4,380,4,0,360,5.1,2,340,15,0,320,26,0,300,42,2
,280,60,2])
opk3("bay",[360,2,1,340,4.1,0,320,15,2,300,36,2,280,56,2,260,76,3])
opv3("bay",[340,7,2])
opv6("bay",[340,13,3,320,6,0])
opv9("bay",[360,35,3,340,20,0,320,8.1,2])
opk9("hfa",[340,10,0,320,15,0,280,31,2,260,50,3])
opk6("hfa",[340,5,0,320,10,0,310,15,0,290,19,0,280,25.1,2,270,37
,2,270,35,0,260,45,3,250,51,2])
opk3("hfa",[320,2.5,1,310,3.6,0,300,6.9,1,290,14,2,280,25,2,270,
33,2,250,52,0,230,72,0])
opv3("hfa",[300,7,2])
opv6("hfa",[300,16,2,280,8,4,270,5.4,4])
opv9("hfa",[300,18,4,280,9,4])
opk9("sch",[600,23,0])
opk6("sch",[550,31,0])
opk9("dgs",[650,5,3,550,17,4,600,13,4])
opk6("dgs",[500,20.5,0])
opk3("dgs",[490,4,4])
opk9("fdn",[320,13,1,300,22,3])
opk6("fdn",[320,10,1])
opk3("fdn",[320,5,4,30,77,1,280,28,4])
```

Figure 3: The file of PROLOG facts which corresponds to the BTX source option data presented in figure 2.

3 Option Valuation Theory

This section presents some results of modern option valuation theory which form a part of OPTRAD's knowledge base. A profound presentation of this theory can be found, for instance, in the textbook of Cox and Rubinstein (1985), while a recent survey on option valuation theory and the empirical evidence has been given by Geske and Trautmann (1986).

From a theoretical point of view, the value of an option is comprised of two components: intrinsic value and time value. Intrinsic value is the financial benefit to be derived if an option is exercised immediately, reflecting the difference between the exercise price and the market price of the underlying asset. Consequently, at the expiration date of an option its value equals always its intrinsic value. Should the stock price on the expiration date be less than the exercise price, a call option owner will not exercise his right to purchase the stock and the option will expire worthless. If, however, the stock price is greater than the exercise price, the call option will be worth the difference between the stock price and the exercise price. Thus, at the expiration date, the value of a call option can be expressed mathematically as

$$C_T = \max\,(S_T - K, 0) \tag{1}$$

where

T	denotes the expiration date,
C_T	denotes the value of the call option at the expiration date T,
S_T	denotes the price of the underlying stock at the expiration date T,
K	denotes the exercise price.

Correspondingly, a put option owner will exercise his right to sell only if it is to his advantage. If the stock price on the expiration date is greater than the exercise price, the put option owner will not exercise his right to sell the stock and the put option will expire worthless. However, should the stock price be less than the exercise price, the put option owner will exercise his right to sell the stock and the put option will be worth the difference between the exercise price and the stock price. Thus, at the expiration date the value of the put option is

$$P_T = \max\,(K - S_T, 0) \tag{2}$$

where P_T denotes the value of the call option at the expiration date T.

During the time remaining before an option expires, the price of the underlying asset can move so as to make the option profitable, or more profitable, to exercise. That is, an option whose intrinsic value is zero can get a positive intrinsic value. The chance that an option will become profitable, or more so, is always greater than zero. Therefore the selling price, or total value, of an option generally exceeds its intrinsic value which corresponds with a strict positive time value. This is especially true for an American-type call option written on a nondividend-paying stock. American-type means that the option can be exercised at any time before maturity. If an option can only be exercised at maturity, it is a European-type option. Like the stock options traded at the Frankfurt Options Exchange, most traded options are of the American type.

According to modern option pricing theory, the time value of a stock option depends on the way in which the future stock price movement is modelled. To specify such a model correctly is a crucial point, but fortunately there exist upper and lower bounds for the time value and total value of an option, respectively, which do not depend on the future stock price movement. Violations of such model-independent value bounds indicate riskless arbitrage opportunities. Riskless arbitrage opportunities in this context are situations that require no initial investment but that yield a positive amount immediately and only nonnegative amounts in the future under all possible circumstances. In the following, three examples for lower bounds will be given.

For American-type call options written on a stock that is not expected to pay dividends or issue rights during the life of the option as well as for completely payout-protected options, Merton (1973a, p. 144) derived what is called the strong European call option lower boundary condition:

$$C \geq \max\left(0, S - Ke^{-rT}\right) \tag{3}$$

where C and S denote the actual option and stock price, respectively, and r is the riskless interest rate. The term "European" stems from the fact that such an option is expected to be held until maturity. Thus, its value is effectively equivalent to an otherwise identical European call option. In well-synchronized markets the violation of (3) indicates that there is a profitable riskless arbitrage opportunity. The trading strategy to profit from a call option violating the European lower bound (3) consists of

- buying the call option,
- selling the stock and
- lending an amount equal to the present value of the exercise price,

and holding this portfolio till expiration. Since short selling is not permitted legally in Germany, this strategy can, however, only be followed by owners of the underlying stock.

Using similar arguments, Galai (1978) derived a lower bound for American-type call options written on dividend-paying stocks and unprotected against dividend payments. In Germany, exchange-traded stock options written before April 1, 1987 were partially protected against dividend payments reducing the stock price: the exercise price was reduced by the amount of the dividend on each ex-dividend date. Nowadays the stock options traded at German options exchanges are completely unprotected against dividend payments, like the options quoted at U.S. exchanges. For these "dividend-unprotected" call options which can be exercised any time until maturity, including the last moment before the stock goes ex-dividend, the early exercise dominance condition for multiple dividends over the life of the option, called pseudo-American lower bound, is

$$C \geq \max\left(0, \max_{i \in I}\left(S - Ke^{-rt_i} - \sum_{j<i} D_j e^{-rt_j}\right)\right) \tag{4}$$

where $I = (0, 1, \ldots, n+1)$, $t_0 = 0$ (the current date), $t_{n+1} = T$, and D_j denotes the amount of dividend (assumed to be non-stochastic) paid at date t_j $(j = 1, 2, \ldots, n)$.

In words, (4) says that the value of an American-type call option not protected for dividends is not less than the maximum of (a) the highest of the European-type call option value for the option computed at each dividend date just before the stock goes ex-dividend, and (b) the European-type call option value assuming that the call option will be held to expiration. If premature exercise is not optimal, the trading strategy to profit from a call option violating the pseudo-American lower bound (4) consists of

- buying the call option,
- selling the stock and
- lending an amount equal to the sum of the present values of the exercise price and the expected dividends,

and holding this portfolio till expiration. If premature exercise is optimal, the trading strategy is analogous except that the position is terminated at some t instead of at T.

The third stock price drift-independent lower value bound to be presented concerns the put option. It is well known that for payout-protected European-type options the relationship between the prices of a put and a call option on the same underlying stock with the same striking price K and the same time to maturity T is given by

$$P = C - S + Ke^{-rT} \tag{5}$$

where P is the current market value of the put option. Any violation of this put-call parity relation, as originated by Stoll (1969), indicates riskless arbitrage opportunities in well synchronized, perfect capital markets. Merton (1973b) generalized this result to the case of American-type options by replacing the equality sign by the "$\geq$" sign in relation (5), which then represents a lower boundary condition (in terms of the call price) for a payout-protected put option:

$$P \geq C - S + Ke^{-rT} \tag{6}$$

Fortunately, the relation (6) remains valid even if the options under consideration are unprotected against dividend payments (see Cox and Rubinstein (1985, p. 152)). The trading strategy to profit from a put option which violates (6) consists of

- buying the put option,
- buying the stock,
- selling the corresponding call option,
- lending an amount equal to the present value of the exercise price,

and holding this portfolio either until the expiration date or the date where the call option is exercised.

In summary, it must be re-emphasized that the European and pseudo-American lower bound for call options (3) and (4), respectively, as well as the call-dependent lower bound for put options (6) are appealing because they imply hypotheses which can be tested without estimating any parameter for the stock or option return distribution.

Observations that the market price of an option did not fall within these bounds would indicate the possible existence of arbitrage opportunities, i. e., above normal profits.

In order to be more specific about the price process for options in a securities market where individuals allocate their wealth to select optimal investments, we need to be more specific about either their risk preferences, their beliefs (about the distribution of stock price changes), or both. Option valuation along the lines of Black and Scholes (1973) avoids restrictive assumptions on individual risk preferences and makes only assumptions on the price movement of the underlying stock.

In their seminal model Black and Scholes (1973) assumed that the stock price changes are continuous with a constant variance of price changes. More precisely, they assumed that the movement of the stock price can be described by a diffusion-type process

$$dS = \mu S\, dt + \sigma S\, dz \tag{7}$$

where μ is the instantaneous expected rate of return on the stock per unit time, σ is the assumed constant instantaneous standard deviation of the return on the stock per unit time, dz is the increment of a standard Gauss-Wiener process. This implies, that the logarithm of asset returns at the end of a period (with finite length) is normally distributed.

The key insight by Black and Scholes (1973) was that it is possible to construct a riskless portfolio involving positions in the stock and the underlying stock, and to avoid the possibility of arbitrage profits, the return on this riskless portfolio must be the riskless interest rate. Under the usual perfect market conditions and the assumption that the stock pays no dividends during the life of the option, the above insight and some mathematical manipulations result in a partial differential equation which must be satisfied by the value of the option. For example the call option must satisfy

$$0 = C_t + \frac{\sigma^2}{2} S^2 C_{SS} + rSC_S - rC \tag{8}$$

where subscripts denote partial derivatives. The solution of this partial differential equation, subject to the terminal condition

$$C_T = \max\,(S_T - K, 0) \tag{1}$$

is the famous Black/Scholes formula for a European-type call option on a nondividend paying stock. It reads:

$$C^{BS} = SN_1(d_1) - Ke^{-rT} N_1(d_2) \tag{9}$$

where

$$d_1 = \{\ln(S/K) + (r + 0.5\sigma^2)T\} \,/\, \sigma\sqrt{T}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$

and $N_1(d)$ is the univariate cumulative normal density function with upper integral limit d.

The value of a call option given by equation (9) can be intuitively interpreted as a weighted difference between the stock price and the present value of the exercise price of the option, in which the weights can only take values between zero and one. When the option is very far "out of the money" (i.e., $S \ll K$) both weights are equal to zero and the the call option value is also equal to zero. When the option is very far "in the money" (i.e., $S \gg K$) both weights have a value of one and the call option has a value equal to the difference between the stock price and the present value of the exercise price. Loosely speaking the weights represent the probability that the option will expire in the money (where the stock price is greater than the exercise price).

Note that the Black/Scholes model value of a call option depends on five model parameters: the actual stock price S, the exercise (or striking) price K, the time to maturity T, the interest rate r and the volatility σ of the stock price. Of these parameters only the last one is not directly observable. An important feature of equation (9) is that the call option value does not directly depend on the expected rate of return of the underlying stock. The reason for this is that the current stock price S already embodies the market expectations about future stock prices.

The assumed absence of income distributions on the underlying security causes the Black/Scholes formula to overstate the value of an American-type call option on a stock with dividend payments during the option's time to expiration. A dividend paid during the option's life reduces the stock price at the ex-dividend instant, and thereby reduces the probability that the stock price will exceed the exercise price at the option's expiration.

Fortunately, there exist two different approaches which are able to consider dividend payments. Both assume that if the stock pays a certain dividend D at the ex-dividend instant t ($t < T$), then the stock price simultaneously falls by a known amount αD. This assumption causes an early exercise probability between zero and one. By means of sophisticated economic reasoning, Roll (1977) succeeded in deriving the following closed-form solution for this valuation problem:

$$\begin{aligned} C^R &= S[N_1(b_1) + N_2(a_1, -b_1; -\sqrt{t/T})] \\ &\quad -Ke^{-rT}[N_1(b_2)e^{r(T-t)} + N_2(a_2, -b_2; -\sqrt{t/T})] \\ &\quad +\alpha De^{-rt}N_1(b_2), \end{aligned} \tag{10}$$

where

$$\begin{aligned} a_1 &= \{\ln(S/K) + (r+0.5\sigma^2)T\}/\sigma\sqrt{T}, & a_2 &= a_1 - \sigma\sqrt{T}, \\ b_1 &= \{\ln(S/S_t^*) + (r+0.5\sigma^2)t\}/\sigma\sqrt{t}, & b_2 &= b_1 - \sigma\sqrt{t}, \end{aligned}$$

and $N_2(a, b; \rho)$ is the bivariate cumulative normal density function with upper integral limits a and b, and correlation coefficient ρ. S_t^* is the ex-dividend stock price determined by

$$C^{BS}(S_t^*, T-t, K) = S_t^* + \alpha D - K \tag{11}$$

above which the option will be exercised just prior to the ex-dividend instant. The left-hand-side of relation (11) denotes the Black/Scholes value of a call option with time to maturity $T - t$, exercise price K and stock price S at time t.

Almost at the same time, Schwartz (1977) provided a numerical method by which the value of an American-type call option an a stock with known dividends can also be calculated. Moreover, this method enables the valuation of American-type call options on dividend-paying stocks as well as American-type put options on dividend-paying stocks. Since American-type options can be exercised at any instant, the boundary conditions must be checked to see if for every possible stock price at each instant, the option is worth more held than exercised (i.e. alive or dead). Thus, if t^- is the instant before exercise and t^+ the instant after, then for put options,

$$P_{t^-} = \max\left(K - S_t, P_{t^+}\right) \tag{12}$$

and for call options,

$$C_{t^-} = \max\left(S_t - K, C_{t^+}\right). \tag{13}$$

If a dividend payment D occurs during the life of the option, the stock price must fall by the previously assumed amount αD to eliminate riskless arbitrage opportunities. If t^- is the instant before the ex-dividend date t and t^+ is the instant after, then

$$S_{t^-} = S_{t^+} + \alpha D. \tag{14}$$

Merton (1973a) was able to demonstrate that call options may be exercised only at the ex-dividend dates, while the exercise boundary condition for put options must be checked at every instant. The latter implies that the valuation of an American-type put option requires the solution of the Black/Scholes partial differential equation (8) (with C replaced by P) subject to the terminal condition (2) and an infinite number of boundary conditions (12) for which no analytic solution have been found. Consequently, Schwartz (1977) devised a numerical method (more precisely, an implicit finite difference approximation scheme) for solving such problems. This numerical method, the valuation formulae of Black/Scholes (1973) and Roll (1977) as well as the boundary conditions described above, form a central part of OPTRAD's knowledge base at the present stage of completion.

4 Recommendation Rules to Purchase or Sell Options or their Underlying Stocks

The main purpose of OPTRAD is to make recommendations to purchase or sell options or their underlying stocks. The detection of options which are seemingly mispriced from the users view forms therefore an important part of OPTRAD. If an option is mispriced, there exists a trading strategy whose profit is abnormal. For a completely riskless trading strategy abnormal profit means profit in excess of the opportunity costs of the initial investment. The uncertain profit of a risky trading strategy is considered abnormal, if, after adjusting for risk by subtracting a risk premium, the expected profit exceeds the opportunity costs of the initial investment where the risk premium is determined by an asset pricing model (the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Model (APM)).

Observed deviations between stock options' market prices and their corresponding model values may occur for the following reasons:

1. The valuation model is not valid.

2. The model parameter are not correctly specified.

3. The markets for the related assets are not well synchronized.

4. Observed market data are inaccurate.

5. The option market is inefficient (allowing abnormal profits for some option traders).

Consequently, the recommendation of trading strategies which guarantee abnormal profits is even in the presence of sophisticated option valuation models a nontrivial task.

```
------------------ OPTRAD - Option Trading Advisor ------------------

Select CALL options which are apparently mispriced with respect to the
       European lower bound
       Pseudo-American lower bound
       Black/Scholes model price
       Roll model price
Select PUT options which are apparently mispriced with respect to the
       Call-dependent lower bound
       Black/Scholes model price (for European-type options)
       Finite-difference model price (for American-type options)
Other: Load data for another date  -  current date: none
       Buy/sell recommendations
       Data Corrections
       BTX Source Data and Text File Corrections
       Fault finding (knowledge base listing)

Select function with cursor keys and <return>              F10 = Main menu
```

Figure 4: Main menu of the advisory dialogue (Screen-dump).

Observed violations of lower boundary conditions for both put and call options, however, can be attributed only to the aforementioned reasons (3)–(5). This enables clearer answers to the question whether the option market under consideration is efficient or not. The main menu of the advisory dialogue (see figure 4) offers therefore, first of all, the selection of options which are mispriced with respect to some model-independent lower value bound. For instance, when the OPTRAD user (say, e.g., a bank) first loads the market data of June 24, 1987 (by selecting the item "`Other: Load data ....`") and then selects the item "`Call-dependent lower bound`", he will get the screen image represented by figure 5. Scrolling in the output file which is displayed through the main window (headed with "`OPTRAD` - Option Trading Advisor") will enable him to select all put options which are apparently mispriced with respect to the call-dependent lower bound from the view of a bank on June 24, 1987.

The attribute "apparently" indicates, that some mispricing signals may be due to the nonsimultaneity of trade of related assets and/or market data inaccuracy. Given accurate data and synchronous trading of stock options and their underlying stock, it would have been possible to implement the aforementioned trading strategy which ensures the abnormal profits or returns (net of all observable opportunity and transaction costs and on a per contract basis) displayed in the column with the heading "Abnormal return". Since, however, neither market synchronization nor data accuracy can be guaranteed, it makes sense to examine whether these mispricing signals occured consistently during the past trading days. Figure 6 reveals the frequency of such mispricing signals during the period from June 5, 1987 to June 24, 1987.

```
Select PUT options which are apparently mispriced                 ┌ Working ┐
with respect to the call-dependent lower bound                     │vhb      │
from the view of a bank on 24.06.87                                └─────────┘

──────────────────── OPTRAD - Option Trading Advisor ──────────────────────────
 Line 1       Col 1      Indent   Insert
Test = pcp    Investor type = bank        Date  = 240687     Price type = A

Stock  Exp.   Striking  S/K-   Call       Put        Abnormal  Critical  Annual
symb.  month  price     Ratio  price      price      return    int.rate  abn.re
-----  -----  --------  -----  --------   --------   --------  --------  ------
 bas    Oct    320.00    OM     10.00TA    20.00G     152.28    0.083
 hwk    Oct    140.00    OM      4.00B     18.00G      94.79    0.097
 hwk    Jul    120.00    AM      5.00       1.00TZ     84.40    0.304         2
 khd    Jul    180.00    AM      4.80       9.00G     133.18    0.320         4
 veb    Jul    300.00    AM     12.00TA     1.00TZ    279.33    0.384         9
 vow    Jul    400.00    AM      9.00TZ     3.00G      72.35    0.114         2

F1:Help F3:Search F4:Subst F5:Copy F6:Move F7:Del F8:ExtEdit F9:ExtCopy F10:End
```

Figure 5: Apparently mispriced put options (with respect to the call-dependent lower bound) observed on June 24, 1987 (Screen-dump).

```
Select PUT options which are apparently mispriced                 ┌ Working ┐
with respect to the call-dependent lower bound                     │vhb      │
from the view of a bank on 24.06.87                                └─────────┘

──────────────────── OPTRAD - Option Trading Advisor ──────────────────────────
Stock  Exp.   Striking 24 23 22 19 18 15 12 11 10  9  5
symb.  month  price     6  6  6  6  6  6  6  6  6  6  6
-----  -----  -------- -- -- -- -- -- -- -- -- -- -- --
 bas    Oct    320.00   *
 con    Oct    340.00                         *
 dai.   Jul   1000.00      *  *  *  *
 drb    Jul    280.00                      *
 drb    Oct    300.00                      *
 hap    Jul    380.00                         *
 hwk    Jul    120.00   *     *
 hwk    Oct    140.00   *
 kfh    Oct    480.00                               *
 khd    Jul    180.00   *     *
 maw    Jul    170.00                      *  *     *
 maw    Oct    170.00                         *     *
 prs    Jul    170.00                         *     *
 sdf    Oct    190.00                               *
 sie    Oct    700.00                               *

F2:Goto line      F3:Search      S-F10:Resize window      F10:End
```

Figure 6: Mispricing signals for put options (with respect to the call-dependent lower bound) observed during the period from June 5, 1987 to June 24, 1987 (Screen-dump).

In OPTRAD a put option is considered as mispriced with respect to the call-dependent lower bound (or more popular: the Put Call Parity (=pcp)), if the put option was apparently mispriced at least "Minimum" times in the last "Frist" days. This flexible mispricing rule is called "mispriced_pcp" in the PROLOG code, and the complete PROLOG code for the selection of mispriced put options with respect to the call-dependent lower bound reads as following:

```
mispriced_pcp(_,_,_,_,X,X,_).

mispriced_pcp(Tag,Monat,Jahr,Frist,Minimum,Anzahl,[T,M,J,_,_|REST]):-
  Minimum>Anzahl,
  innerhalb_der_frist(Frist,Tag,Monat,Jahr,T,M,J),
  Anzahl2=Anzahl+1,
  mispriced_pcp(Tag,Monat,Jahr,Frist,Minimum,Anzahl2,REST).

mispriced_pcp(Tag,Monat,Jahr,Frist,Minimum,Anzahl,[_,_,_,_,_|REST]):-
  Minimum>Anzahl,
  mispriced_pcp(Tag,Monat,Jahr,Frist,Minimum,Anzahl,REST).
```

It is reasonable to specify the parameters "Minimum" and "Frist" such that from the apparently mispriced put options presented in figure 6, only the put option written on DAIMLER stocks (stock symbol: dai) with expiration date July 15, 1987 and exercise price DM 1000,– is considered as mispriced.

In a similar way stock options are considered as mispriced with respect to other model-independent lower bounds or with respect to some model value. However, although most parameters of the option valuation models previously discussed are in principle observable without error, the future volatility of the underlying stock is unknown. It must be estimated either from the historical or the implied volatility. The latter obtains by equating the option's actual market price to the model value, and solving (numerically) for the only unobservable parameter, the volatility. In general it is difficult to estimate or predict the future volatility precisely. Therefore OPTRAD considers an option as apparently mispriced with respect to some model value only if the deviation between the observed market price and the corresponding model value is sufficiently large. For example, according to the PROLOG rule "bsmodi" presented in figure 7, options are considered as mispriced with respect to the Black/Scholes model if the relative deviation $|BS - PR|/PR$ between the market price PR and the model value BS is larger than "Filter" percent.

Although all mispriced options detected by OPTRAD are candidates for a buy or sell recommendation, not all mispriced options can be bought or sold because of institutional and financial restrictions.

For instance, since short selling is not permitted legally in Germany, a call option which is overpriced (i.e., when the market price is above the model value) can be written or sold only by owners of the underlying stock or the call option, respectively. Clearly, OPTRAD considers only those mispriced options to be good buys or sells whose associated trading strategy is compatible with institutional and financial restrictions, the actual portfolio position, and last but not least: the investor's risk preference.

```
bsmodl(Name,Anl,PA,Filter,K,Art,Vol,T,AK,Laufzeit,Jahre,[BP,PR,P_ART|P]):-
  bsmodl(Name,Anl,PA,Filter,K,Art,Vol,T,AK,Laufzeit,Jahre,P),!,
  kurs_ok(PA,P_ART),
  zinssatz(Anl,Zins,_),
  BP>0,
  PR>0,
  R_hoch_mt=exp(-Jahre*ln(Zins)),
  E_hoch_mrt=exp(-Jahre*Zins),
  V=(ln(AK/BP)+(Zins*Jahre)+(Vol*Vol*Jahre/2))/(Vol*sqrt(Jahre)),
  normalvert_1(V,Delta),
  V2=V-(Vol*sqrt(Jahre)),
  normalvert_1(V2,Delta2),
  BSc=AK*Delta-(BP*E_hoch_mrt*Delta2),
  bs_fehlbewertet(Art,BSc,BS,AK,BP,PR,R_hoch_mt,Filter),
/*    Abweichung=(BS-PR)/PR,    */
  Gamma=exp(-V*V/2)/(2.5066283*AK*Vol*sqrt(Zins)),
  Theta=(AK*Vol*exp(-V*V/2)/(5.0132566*sqrt(Jahre)))+(BP*R_hoch_mt*log(Zins)*Delta2),
  SdK=AK/BP,finde_typ(SdK,Typ),kzt(P_ART,Zusatz,_),
  laufzeit_tage(Laufzeit,Tage),
  datum(Heute),
  Endtag=Heute+Tage,
  endtag_monat(Endtag,Monat),
  openappend(save_file,Name),
  writedevice(save_file),
  writef("  %4   %3   %8.2   %3   %7.2%2  %8.2  %5.2  %7.4  %9.2  %8.2  %1",
        K,Monat,BP,Typ,PR,Zusatz,BS,Delta,Gamma,Theta,AK,T),
  nl,
  writedevice(screen),
  closefile(save_file),
  !,
  fail.

bsmodl(_,_,_,_,_,_,_,_,_,_,_,_).
```

Figure 7: PROLOG code of the mispricing rule "bsmod1".

5 The Choice of the Development Language

At the beginning of the project we looked for an appropriate programming environment to write the prototype. PROLOG seemed to be a more powerful tool than procedural programming languages like PASCAL or FORTRAN for developing a decision support system.

Using PROLOG, the user is free to concentrate on issues of knowledge representation and acquisition rather than search strategies. One can naturally develop a top down algorithm. This allows rapid adjustment of decision rules in OPTRAD as recommended by brokers and financial researchers. Typically the expert does not know a priori how he thinks, and the system inevitably develops somewhat by trial and error.

However, during the process of system development we recognized that for some tasks of OPTRAD the procedural language PASCAL was the better choice. The BTX communication part and the conversion programs were exclusively written in PASCAL. The first intention to write these programs in PROLOG was dropped during the development. The BTX communication software works at assembler and BIOS (Basic Input Output System) level for communication with the BTX decoder. This is the reason for problems that would occur in case of a PROLOG implementation. Manufacturers of BTX decoders and BTX communication software told us the same experience.

Moreover, the implementation of a complete linear conversion algorithm, that is not based on any rules, can be done much faster in PASCAL than in PROLOG. For instance the following short program cut-out shows the PASCAL statements to eliminate superfluous blanks in BTX source files:

```
while not eof(FI) do
  begin
    readln (FI,SI2);
    if (SI1[length(SI1)] = ' ') and (SI2[1] = ' ') then
      delete (SI2,1,1);
    while pos('  ',SI2) > 0 do
      delete (SI2,pos('  ',SI2),1);
    writeln (FO,SI2);
    SI1 := SI2;
  end;
```

This could not have been implemented in PROLOG so fast and easy. Furthermore, the valuation of put options by the numerical methods mentioned above is very difficult to realize in PROLOG. Therefore a fast and powerful system has to include interfaces to programming languages like PASCAL and FORTRAN. Numerical procedures could then be implemented in these languages while the main program could be realized in an expert system shell or PROLOG.

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