External Performance Attribution with the Exponential Performance Measure

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Abstract

A reasonable requirement for a performance measure is that the performance measured can be split up in a timing and a selectivity component. Unfortunately, members of the class of admissible performance measures violate in general this requirement. But there is at least one exception: a member of the class of positive admissible performance measures which we call *exponential performance measure*. This measure assumes that the uninformed investor possesses an exponential utility function exhibiting constant absolute risk aversion.

Using a local market model we show how timing and selectivity affect performance. We can express analytically the dependence of performance on timing and selectivity for a quadratic and an exponential utility function. This enables us to isolate timing and selectivity relying only on return data. This *external* performance attribution contrasts with the internal performance attribution which uses in addition portfolio weight information. Since timing and selectivity cause deviations from the passive strategy, performance can be compared with the risk of a passive strategy. This allows a ranking of mutual fund performance.

Return data of 17 German mutual funds from 1975 to 1994 indicate that in the past German mutual fund portfolio managers were good stock pickers and not that good market timers.

1 Introduction

Traditional performance measurement assumes (1) a buy and hold-strategy implying a constant portfolio beta and (2) an efficient market index to compute the appropriate betas. Performance measures like Jensen's (1968) *alpha* or Treynor's (1965) *reward to volatility ratio* compare the *characteristic line* of a portfolio with a *passive strategy* that consists of fixed fractions of the investor's funds invested in the market proxy and the risk free asset. The characteristic line results from the linear regression of the portfolio's excess rate of return on the excess rate of return of the market proxy used. The slope of the line corresponds to the value of beta. The intercept equals Jensen's alpha. In case of *market timing* the portfolio manager adjusts the portfolio's composition and consequently the portfolio beta to anticipated market movements. A positive correlation between portfolio beta and market excess rate of return yields an alpha which is biased downwards. Furthermore, an inefficient market proxy may lead to alphas which are biased upwards as well as downwards.

Grinblatt and Titman (1989) were the first to propose a performance measure that assigns positive performance to portfolio managers with timing abilities. Their *positive period weighting measure* has the same data requirements as Jensen's alpha but identifies correctly informed investors as positive performers. Glosten and Jagannathan (1994) embed the performance measurement issue in a broader framework. They show that valuing performance is equivalent to valuing particular contingent claims on an index portfolio. Like Grinblatt and Titman (1989), Glosten and Jagannathan (1994) point out that performance measurement needs a valuation model with *positive* state price densities. The latter property guarantees that the service provided by the portfolio manager, namely creating "costlessly" a call option on the index, has a positive value. Since in incomplete markets there are numerous candidate price densities (or period weighting measures, to use the Grinblatt and Titman (1989) terminology), Glosten and Jagannathan (1994) assume the existence of an representative passive investor whose *intertemporal marginal rate of substitution* (IMRS) characterizes the state price density.

Chen and Knez (1996) propose an axiomatic approach to performance measurement which parallels the axiomatic approach of arbitrage models for securities valuation. According to that approach any *admissible performance measure* (APM) should satisfy four minimal conditions: it assigns zero performance to passive strategies and it is linear, continuous, and nontrivial. Such an APM exists if and only if the securities market obeys the *law of one price*. Using a *positive* APM (PAPM) is equivalent to the assumption that there are *no arbitrage* opportunities for uninformed investors. Each APM is uniquely representable by a so-called *stochastic discount factor* which can be identified with the IMRS of Glosten and Jagannathan (1994). The performance measures of Grinblatt and Titman (1989) and Glostan and Jagannathan (1994) belong to the PAPM-class.

To assess the portfolio manager's specific abilities (timing and selectivity) we have to assume a return generating model for the portfolio rates of return. A reasonable requirement for a performance measure is that the performance measured can be split up in a timing and a selectivity component. Unfortunately, members of the APM-class violate in general this requirement. But there is at least one exception: a member of the PAPM-class which we will call *exponential performance measure* (EPM). This measure assumes that the uninformed investor possesses an exponential utility function exhibiting constant absolute risk aversion (CARA).

Using a local market model we show how timing and selectivity affect performance. We can express analytically the dependence of performance on timing and selectivity for a quadratic and an exponential utility function. This enables us to isolate timing and selectivity relying only on return data. This so-called *external* performance attribution contrasts with the *internal* performance attribution as proposed by, e.g., Grinblatt and Titman (1989) and Heinkel and Stoughton (1997). Like Grinblatt and Titman (1993), the latter paper uses in addition portfolio weight information. Since timing and selectivity cause deviations from the passive strategy, performance can be compared with the risk of a passive strategy. This allows a ranking of mutual fund performance.

This paper is organized as follows: In section 2 we discuss the class of APMs. Section 3 introduces the EPM. Section 4 presents empirical results concerning timing and selectivity of German mutual funds. Section 5 shows the connection between external and internal performance measurement. Section 6 concludes with a summary.

2 Admissible Performance Measures

Assume that the securities market is incomplete and that the securities' excess rate of return r_P is a mean-square integrable random variable on some probability space $\{\Omega, F, Pr\}$. Let L^2 be the linear space of mean-square integrable random variables on $\{\Omega, F, Pr\}$. According to Chen and Knez (1996) an admissible performance measure (APM) is a function $\alpha(\cdot) : L^2 \rightarrow \mathbb{R}$, fulfilling the following four conditions:

(C1) *Zero performance of passive strategies:* Every portfolio composed according to a passive strategy shows a zero performance.

(C2) *Linearity:* The performance of combined portfolios is the weighted performance of the subportfolios.

(C3) *Continuity:* If the returns produced by two managed portfolios are arbitrarily close, the performance values assigned to them will also be arbitrarily close.

(C4) *Non-triviality:* Single assets do not have a zero performance. If a portfolio manager achieves an additional rate of return that can be identified with an asset rate of return, a non-zero performance is shown.

A *positive* admissible performance measure (PAPM) satisfies in addition condition

(C5) *Positivity:* The performance measure shows a positive performance if the portfolio manager acts on private information.

An APM exists if and only if the securities market obeys the *law of one price.* A PAPM exists if and only if there are *no arbitrage* opportunities in the market. Chen and Knez (1996) show that performance evaluation is generally quite *arbitrary.* They prove that for each APM there is some *state price density,* sometimes called *stochastic discount factor,* $w \in L^2$, such that $\alpha(\cdot)$ has the representation

$$\alpha(r_P) = \mathrm{E}(w \cdot r_P) \tag{1}$$

for all excess rates of return $r_P \in L^2$, where w satisfies $\mathrm{E}(w \cdot R_P) = k \in \mathbb{R}$ for all rates of return R_P.

2.1 Performance Measurement in a Local Market Model

To specify an APM we look at an investor who follows a passive strategy. From his perspective the investor holds an efficient portfolio when splitting his funds into the risk free asset and the tangency portfolio according to his risk aversion. The latter needs not to be the global market portfolio. Therefore, this performance measure is not affected by mismeasurement resulting from a misspecified market proxy.

Let x denote the fraction of the investor's funds invested in the tangency portfolio denoted by M, the investor's rate of return R from a passive strategy reads as follows:

$$R(x) \quad = \quad x \cdot R_M + (1-x) \cdot r_f = x \cdot r_M + r_f. \tag{2}$$

With the convention

$$w \quad \equiv \quad \frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}, \tag{3}$$

the benchmark performance must be zero:

$$\mathrm{E}(w \cdot r_M) \quad = \quad \mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)} \cdot r_M\right) \quad \stackrel{!}{=} \quad 0, \tag{4}$$

consistent with condition (C1).

Chen and Knez (1996) point out that an APM yields the normalized marginal utility of an investor who buys one marginal unit of the mutual fund and sells one marginal unit of the index portfolio. With $R(y) = y \cdot r_P + (x^* - y) \cdot r_M + r_f$ we have:

$$\begin{aligned}
&\frac{\partial}{\partial y}\mathrm{E}\left(\frac{u\big(R(y)\big)}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\right)\Big|_{y\,=\,0} \qquad (5)\\
=\;& \mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\Big|_{y\,=\,0} \cdot (r_P - r_M)\right)\\
=\;& \underbrace{\mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\Big|_{x^*} \cdot r_P\right)}_{=\,\alpha(r_P)} - \underbrace{\mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\Big|_{x^*} \cdot r_M\right)}_{=\,0}.
\end{aligned}$$

The following two assumptions are sufficient to represent an APM as the sum of the selection component and a potentially biased timing component.

(A1) The portfolio excess rates of return follow the return generating process

$$r_{Pt} = \beta_{Pt} \cdot r_{Mt} + \epsilon_{Pt}, \tag{6}$$

where the excess rate of return of the benchmark r_M and the portfolio residual ϵ_P are not correlated: $\mathrm{Cov}(r_M, \epsilon_P) = 0$.

(A2) Portfolio beta and benchmark excess rate of return are jointly normally distributed.

Definition 1: A return generating model is called a *local market model* if the excess rate of return of each portfolio considered in the model is generated according to assumption (A1).

Definition 2: An investor is said to have *timing ability* if $\mathrm{Cov}(\beta_P, r_M) > 0$ given that the excess rates of return are generated according to a local market model.

Definition 3: An investor is said to have *selectivity* if $\mathrm{E}(\epsilon_P) > 0$ given that the excess rates of return are generated according to a local market model.

Timing ability and selectivity defined according to definitions 2 and 3 are affected not only by the information processing ability but also by the aggressiveness of the portfolio manager. As Heinkel and Stoughton (1997) point out, both performance components can be thought of as the product of the manager's information processing ability and his risk tolerance. Therefore, the total performance is in some sense a price for the manager's superior information *and* his skills in using it. It reflects the marginal value of the mutual fund investment to the investor. But because timing ability and selectivity are affected by the manager's aggressiveness in the same way, the ratio of both performance components is independent of the manager's risk tolerance.[1]

[1] To separate measures of information quality from measures of aggressiveness, Jensen (1972), Bhattacharya and Pfleiderer (1983), and Admati et al. (1986) use the quadratic regression approach of Treynor and Mazuy (1966) relying only on return data. These studies model the fund manager's portfolio choice by utilizing the standard normally distributed signal methodology and assuming constant absolute risk aversion of the portfolio manager. Our approach does not model the fund manager's portfolio choice.

Proposition 1: *Assume that assumptions (A1) and (A2) hold. Then each APM $\alpha(\cdot)$ has the representation*

$$\alpha(r_P) \;=\; \left(1 + \mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right)\right) \cdot \underbrace{\mathrm{Cov}(\beta_P, r_M)}_{timing} \;+\; \underbrace{\mathrm{E}(\epsilon_P)}_{selectivity} \;. \qquad (7)$$

PROOF: With assumption (A1) an APM has the representation

$$\begin{aligned}\alpha(r_P) \;&=\; \mathrm{E}(w \cdot r_P) \;=\; \mathrm{E}(w \cdot \beta_P \cdot r_M) + \underbrace{\mathrm{E}(w)}_{=1} \cdot \mathrm{E}(\epsilon_P) \\ &=\; \mathrm{Cov}(\beta_P, w \cdot r_M) + \mathrm{E}(\beta_P) \cdot \underbrace{\mathrm{E}(w \cdot r_M)}_{=0} + \mathrm{E}(\epsilon_P).\end{aligned}$$

Stein's lemma together with assumption (A2) gives

$$\begin{aligned}\alpha(r_P) \;&=\; \mathrm{E}\left(\frac{\partial}{\partial r_M}(w \cdot r_M)\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P) \\ &=\; \left(1 + \mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right)\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P).\end{aligned}$$

□

2.2 Special APMs in the Local Market Model

Grinblatt and Titman (1989) suggest a PAPM which they call the *positive period weighting measure.* The corresponding maximum likelihood estimator is defined as follows:

$$\widehat{\alpha}(r_P) \;\equiv\; \sum_{t=1}^{T} w_t \cdot r_{Pt}, \qquad (8)$$

where the so-called weights w_t are subject to the constraints

(P1) $\quad \operatorname{p\,lim}_{T\to\infty} \sum_{t=1}^{T} w_t \cdot r_{Mt} = 0,$

(P2) $\quad \sum_{t=1}^{T} w_t = 1,$

(P3) $\quad w_t > 0.$[2]

[2]In addition, a technical condition requires the period weights to converge sufficiently fast.

The period weight w_t, $t = 1, \ldots, T$, times the number of observations T $(T \cdot w_t)$ is a realization of w. Maximizing expected utility of a passive investor yields period weights which simply have to be normalized. A positive marginal utility ensures positive period weights. The positive period weighting measure is positively related to timing and/or selection abilities. But performance does not equal the sum of timing and selectivity. However, positive period weighting measures within the local market model assign a zero performance to single assets:

$$\alpha(r_i) \;=\; \mathrm{E}(w \cdot r_i) = \beta_i \cdot \mathrm{E}(w \cdot r_M) + \mathrm{E}(\epsilon_i) = 0. \tag{9}$$

Therefore, any passive strategy shows a zero performance. Hence, the positive period weighting measures belong to the PAPM-class.

Jensen's Alpha: A Non-positive APM

Jensen's alpha turns out as a special APM for the following period weights:

$$w_t = \frac{1}{T} \cdot \left(1 - \frac{\widehat{r}_M}{\widehat{\sigma}_M^2} \cdot (r_{Mt} - \widehat{r}_M)\right) \;\Rightarrow\; \widehat{\alpha}(r_P) = \widehat{r}_P - \widehat{\beta}_P \cdot \widehat{r}_M = \widehat{\alpha}_P. \tag{10}$$

A negative selectivity is shown with excess rates of return reaching a certain level as these returns get negative weights. Grinblatt and Titman (1989) conclude that the linear weights fitting with Jensen's alpha are consistent with a quadratic utility function: $\alpha_P = \alpha^{\mathrm{qu}}(r_P)$. This utility function is crucial since is reflects increasing absolute risk aversion and implies negative marginal utility. Jensen's alpha fails to fulfill the condition of a positive APM since the characteristic line fails to correctly represent the option-like character of timing activities.[3]

[3]Dybvig and Ingersoll (1982) point out that for risky payoffs P the weights leading to Jensen's alpha give a valuation rule according to the standard capital asset pricing model:

$$\begin{aligned} P_0 &= \frac{\mathrm{E}(w \cdot P)}{1 + r_f} \quad \text{where} \quad w = 1 - \frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \big(r_M - \mathrm{E}(r_M)\big) \\ \Rightarrow \quad \mathrm{E}(R_P) &= r_f + \beta_P \cdot \big(\mathrm{E}(R_M) - r_f\big) \quad \text{where} \quad R_P \equiv \frac{P - P_0}{P_0}. \end{aligned}$$

This valuation rule allows arbitrage opportunities. For example, index call options with a sufficiently high strike have negative prices. This is due to the non-positivity of the valuation operator.

The timing bias of Jensen's alpha amounts to

$$\mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right) \cdot \mathrm{Cov}(\beta_P, r_M) \;=\; -\frac{\mathrm{E}^2(r_M)}{\sigma_M^2} \cdot \mathrm{Cov}(\beta_P, r_M). \tag{11}$$

Cumby and Glen's Measure: A Positive APM

Cumby and Glen (1990) were the first using a power utility function to specify the period weights. While a positive or negative performance is correctly indicated, usually a timing bias still exists.[4] Based on the power utility function

$$u(R) = \frac{1}{1-\theta} \cdot R^{1-\theta}, \tag{12}$$

with constant relative risk aversion θ and rate of return $R(x) = 1 + x \cdot r_M + r_f$, the timing bias amounts to

$$\mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right) \cdot \mathrm{Cov}(\beta_P, r_M) \;=\; -\theta \cdot \mathrm{E}\big(R(x^*) \cdot w \cdot r_M\big) \cdot \mathrm{Cov}(\beta_P, r_M). \tag{13}$$

3 The Exponential Performance Measure

Performance measures relying on the marginal utility of a passive investor parallel asset valuation rules based on the preferences of the representative investor. For example, Brennan (1979) derives for assets with end-of-period payoff P the valuation rule

$$P_0 \;=\; \frac{1}{1+r_f} \cdot \mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)} \cdot P\right). \tag{14}$$

We now assume a passive investor exhibiting exponential utility with utility function $u(R) = -\exp\{-a \cdot R\}$ with CARA a. Assuming normally distributed rates of return, the optimal passive strategy x^* is computed by maximizing the certainty equivalent:

[4]Cumby and Glen (1990) and Grinblatt and Titman (1994) present empirical results for US mutual funds. Wittrock (1995, pp. 311–314) and Wittrock and Steiner (1995) analyze German mutual funds.

$$x^* = \arg\max_x \ \mathrm{E}\big(R(x)\big) - \frac{a}{2} \cdot \mathrm{Var}\big(R(x)\big), \tag{15}$$

where $R(x) = x \cdot r_M + r_f$. The optimal fraction of funds invested in the tangency portfolio is

$$x^* = \frac{\mathrm{E}(r_M)}{a \cdot \sigma_M^2}. \tag{16}$$

The ratio of marginal utility to expected marginal utility represents the IMRS. With an exponential utility function and normally distributed rates of return this ratio computes as follows:

$$\begin{aligned} \frac{\partial}{\partial R} u(R)\Big|_{x^*} &= a \cdot \exp\{-a \cdot r_f\} \cdot \exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot r_M\right\} \\ \Rightarrow \quad \mathrm{E}\left(\frac{\partial}{\partial R} u(R)\right)\Big|_{x^*} &= a \cdot \exp\{-a \cdot r_f\} \cdot \exp\left\{-\frac{\mathrm{E}^2(r_M)}{2 \cdot \sigma_M^2}\right\} \\ \Rightarrow \quad \frac{\partial u(R)/\partial R}{\mathrm{E}\big(\partial u(R)/\partial R\big)}\Big|_{x^*} &= \exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \left(r_M - \frac{\mathrm{E}(r_M)}{2}\right)\right\}. \end{aligned} \tag{17}$$

Therefore, a performance measure based on the state price density

$$w(r_M) = \exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \left(r_M - \frac{\mathrm{E}(r_M)}{2}\right)\right\} \tag{18}$$

belongs to the Glosten and Jagannathan (1994) IMRS-class of performance measures.

Definition 5: *A PAPM based on state price density (18) is called exponential performance measure (EPM), denoted by* $\alpha^{\mathrm{eu}}(r_P)$.

Jensen's alpha may lead to mismeasurement since the stochastic discount factor consistent with Jensen's alpha is a linear approximation of the exponential stochastic discount factor. When taking the adjusted exponential weights

$$\exp\left\{\frac{\widehat{r}_M^2}{2\widehat{\sigma}_M^2}\right\} \cdot w(r_{Mt}) = \frac{1}{T} \cdot \exp\left\{-\frac{\widehat{r}_M}{\widehat{\sigma}_M^2} \cdot (r_{Mt} - \widehat{r}_M)\right\} \tag{19}$$

into consideration and developing a Taylor series at $\widehat{r}_M$, we obtain that the first part of the series equals exactly the period weights consistent with Jensen's alpha.[5]

3.1 Performance equals the Sum of Timing and Selectivity

The following proposition ensures that the EPM $\alpha^{\mathrm{eu}}(r_P)$ is not biased through timing.

Proposition 2: *The EPM can be decomposed in an unbiased timing component and a selectivity component:*

$$\alpha^{\mathrm{eu}}(r_P) \quad = \quad \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P). \tag{21}$$

[5]In contrast to the quadratic state price density (10), the exponential state price density (18) guarantees positive option values. Therefore, the latter assigns positive values to timing abilities. Let $C = \max\{M - K, 0\}$ denote the payoff of a European call option written on a market index M with strike K. With the convention $X \equiv M/M_0$ and defining the index rate of return—in contrast to Brennan (1979)—by:

$$R_M \quad = \quad \ln X + \frac{\sigma_M^2}{2} \quad \text{where} \quad R_M = r_M + r_f$$

we surprisingly get the Black–Scholes value of the end-of-period payoff C:

$$\begin{aligned}
& e^{-r_f} \cdot \mathrm{E}\big(w(r_M) \cdot C\big) \qquad\qquad (20)\\
= \; & e^{-r_f} \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma} \int_K^{\infty} \left(M_0 - \frac{K}{X}\right) \cdot \exp\left\{ - \frac{\left(\ln X - r_f + \frac{\sigma_M^2}{2}\right)^2}{2\sigma^2} \right\} dX \\
= \; & M_0 \cdot \frac{1}{\sqrt{2\pi}} \int_{\frac{1}{\sigma_M} \cdot \left(\ln \frac{K}{M_0} - r_f + \frac{\sigma_M^2}{2}\right)}^{\infty} \exp\left\{ - \frac{(y - \sigma_M)^2}{2} \right\} dy \\
& - e^{-r_f} \cdot K \cdot \frac{1}{\sqrt{2\pi}} \int_{\frac{1}{\sigma_M} \cdot \left(\ln \frac{K}{M_0} - r_f + \frac{\sigma^2}{2}\right)}^{\infty} \exp\left\{ - \frac{y^2}{2} \right\} dy \\
= \; & M_0 \cdot \mathrm{N}\left(\frac{\ln \frac{M_0}{K} + r_f + \frac{\sigma^2}{2}}{\sigma} \right) - e^{-r_f} \cdot K \cdot \mathrm{N}\left(\frac{\ln \frac{M_0}{K} + r_f - \frac{\sigma^2}{2}}{\sigma} \right).
\end{aligned}$$

PROOF: The performance assigned to the benchmark is zero:

$$\begin{aligned}
\mathrm{E}\big(w(r_M)\cdot r_M\big) &= \mathrm{E}\left(\exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2}\cdot\left(r_M-\frac{\mathrm{E}(r_M)}{2}\right)\right\}\cdot r_M\right)\\
&= \frac{1}{\sqrt{2\pi}\cdot\sigma_M}\int_{-\infty}^{\infty} r_M\cdot\exp\left\{\frac{r_M^2}{2\sigma_M^2}\right\}d\,r_M = 0.
\end{aligned}$$

Therefore, the timing component is not biased:

$$\mathrm{E}\left(\frac{\partial}{\partial r_M}w(r_M)\cdot r_M\right) = -\frac{\mathrm{E}(r_M)}{\sigma_M^2}\cdot\underbrace{\mathrm{E}\big(w(r_M)\cdot r_M\big)}_{=0} = 0.$$

□

Passive investors follow a buy and hold-strategy that keeps the portfolio composition unchanged. The performance of passive strategies is zero. Timing abilities resemble options since perfect timing achieves the maximum from the market return and the risk free asset return. Merton (1981) points out that in this case portfolio payoffs are identical to a portfolio that combines index calls with the risk free asset. A buy and hold-strategy cannot duplicate the index option due to the lack of continuous portfolio changes. Thus, the evaluation is done as if the market is complete. This is achieved with an assumption on preferences.

We now like to show why the EPM decomposes simply in a timing and a selectivity component. Recall that the state price density $w(r_M)$ might be identified with the IMRS of a representative investor. Furthermore, the marginal rate of return from a passive strategy corresponds to the excess rate of return of the tangency portfolio. The timing bias is zero:

$$\begin{aligned}
\mathrm{E}\left(\frac{\partial}{\partial r_M}w(r_M)\cdot r_M\right) &= \mathrm{E}\Bigg(\frac{\partial}{\partial r_M}\underbrace{\frac{\partial u(R)/\partial R}{\mathrm{E}\big(\partial u(R)/\partial R\big)}}_{=w(r_M)}\cdot\underbrace{\frac{\partial R}{\partial x}}_{=r_M}\Bigg)\Bigg|_{x^*} \qquad (22)\\
&= \mathrm{E}\Bigg(\underbrace{\frac{\partial}{\partial R}\frac{\partial u(R)/\partial R}{\mathrm{E}\big(\partial u(R)/\partial R\big)}}_{=-a\cdot\frac{\partial u(R)/\partial R}{\mathrm{E}(\partial u(R)/\partial R)}}\cdot\underbrace{\frac{\partial R}{\partial r_M}}_{=x^*}\cdot\frac{\partial R}{\partial x}\Bigg)\Bigg|_{x^*}\\
&= -\frac{a\cdot x^*}{\mathrm{E}\big(\partial u(R)/\partial R\big)}\cdot\underbrace{\mathrm{E}\left(\frac{\partial}{\partial x}u\big(R(x)\big)\right)\Bigg|_{x^*}}_{=0}.
\end{aligned}$$

CARA implies that the second derivative of the exponential utility function is proportional to the first derivative. Therefore, the major benefit of the EPM is due to the CARA property.

3.2 External Performance Attribution

The EPM allows to isolate the timing component and the selectivity component relying only on return data.

Definition 5: Isolating the timing component from the selectivity component using only return data is called *external performance attribution.*

Proposition 3: *Assume that an uninformed investor exhibits CARA. Then the timing component of the performance is proportional to the difference of the EPM* $\alpha^{eu}(r_P)$ *and Jensen's alpha* $\alpha^{qu}(r_P)$ *given by:*

$$\mathrm{Cov}(\beta_P, r_M) = \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}. \qquad (23)$$

The selectivity component of the performance computes as follows:

$$\mathrm{E}(\epsilon_P) = \alpha^{\mathrm{eu}}(r_P) - \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}. \qquad (24)$$

PROOF: The EPM and Jensen's alpha have the representation:

$$\begin{aligned} \alpha^{\mathrm{eu}}(r_P) &= \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P); \\ \alpha^{\mathrm{qu}}(r_P) &= \left(1 - \frac{\mathrm{E}^2(r_M)}{\sigma_M^2}\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P). \end{aligned}$$

Therefore, we can isolate the timing component and the selectivity component:

$$\begin{aligned} \mathrm{Cov}(\beta_P, r_M) &= \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)} \\ \Rightarrow \ \mathrm{E}(\epsilon_P) &= \alpha^{\mathrm{eu}}(r_P) - \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}. \end{aligned}$$

□

Like Jensen's alpha, most APMs do not allow a ranking of mutual fund performance. In this manner, a second mutual fund with a higher performance might exist that requires a higher average portfolio beta. The average portfolio beta can be computed without knowing the portfolio composition and can further be used to adjust the obtained performance.

Proposition 4: *The EPM divided by the average beta*

$$\mathrm{E}(\beta_P) = \frac{\mathrm{E}(r_P) - \alpha^{\mathrm{eu}}(r_P)}{\mathrm{E}(r_M)} \tag{25}$$

allows a ranking of mutual fund performance. We call the ratio of expected fund excess rate of return divided by the average beta the Treynor ratio with average beta.

PROOF: When using the EPM, the expected portfolio rate of return can be decomposed as follows:

$$\begin{aligned}
\mathrm{E}(r_P) &= \underbrace{\mathrm{E}(\beta_P) \cdot \mathrm{E}(r_M)}_{\text{benchmark return}} + \underbrace{\mathrm{Cov}(\beta_P, r_M)}_{\text{timing}} + \underbrace{\mathrm{E}(\epsilon_P)}_{\text{selectivity}} \\
&= \underbrace{\mathrm{E}(\beta_P) \cdot \mathrm{E}(r_M)}_{\text{benchmark return}} + \underbrace{\alpha^{\mathrm{eu}}(r_P)}_{\text{performance}} .
\end{aligned}$$

Dividing by $\mathrm{E}(\beta_P)$ yields:

$$\frac{\alpha^{\mathrm{eu}}(r_P)}{\mathrm{E}(\beta_P)} = \frac{\mathrm{E}(r_P)}{\mathrm{E}(\beta_P)} - \mathrm{E}(r_M).$$

□

With positive performance and positive average portfolio beta, the obtained mutual fund excess rate of return per unit of average systematic risk exceeds the one of a passive strategy. The higher the difference the more benefitial is the portfolio management to the investor.

As mentioned above, the performance measured by the EPM is proportional to the risk tolerance of the portfolio manager. In the standard exponential-normal framework of portfolio choice all portfolio weights are proportional to the manager's risk tolerance. Since the portfolio beta ist the weighted sum of asset betas, the manager's risk tolerance has an identical impact on the average portfolio beta.

Therefore, the Treynor ratio with average beta is *independent* from the manager's risk tolerance. Consequently, mutual fund performance measured by the EPM and divided by the average beta allows a ranking of the information processing abilities of the portfolio managers. Therefore, we are able to isolate their information processing abilities *without* knowing the managers' risk tolerances.

4 Empirical Performance Estimates

4.1 Data

The analysis is based on monthly return data of 17 German equity mutual funds from 1975 to 1994.[6] The computed rates of return assume that after-tax payouts are reinvested in the mutal fund.[7] The risk free interest rate is assumed to be the 1-month FIBOR or the 1-month money market rate, respectively; we used DAFOX, CDAX, and DAX returns as proxies of market returns. While the first two market proxies include all listed stocks at the Frankfurt stock exchange the DAX includes 30 German blue chips that represent 75 % of the German stock transactions.

[6]Recently, the number of mutual funds has substantially increased but a complete time series exists only for these mutual funds. According to Brown et al. (1992) empirical analysis is distorted by a survivorship bias resulting from defaulted mutual funds. This effect does not apply here since the only mutual fund that disappeared was subject to reconstruction—not default.

[7]In all tables and figures rates of return are annualized based on discretely compounded rates of return.

The backwards calculation of the DAFOX for the time before its introduction differs from the one for the CDAX and the DAX because those are connected with the Hardy index and the index of the 'Börsenzeitung', respectively. Therefore, the DAFOX which was created for empirical research is closest to the market portfolio.[8]

4.2 Traditional Performance Measurement

Table 1 presents the results of traditional performance measurement based on Jensen's alpha and Treynor's ratio where the degree of diversification equals the coefficient of determination R^2.

Although all mutual funds considered here earned a positive risk premium, the average performance is nearly zero (−0.08 %) when using the DAFOX as the benchmark. When using the CDAX as market proxy, both alphas and betas of all mutual funds increase. When using the DAX Jensen's alphas are substantially higher while betas are lower. This leads to higher Treynor ratios.

[8]See Göppl and Schütz (1992). Mutual fund price data were provided by the 'BVI Bundesverband Deutsche Investment-Gesellschaften e.V.' DAFOX quotations and CDAX quotations came from the 'Karlsruher Kapitalmarktdatenbank'. DAX quotations were available from the 'Deutschen Börse AG'. Money market rates are taken from the monthly reports of the 'Deutsche Bundesbank'. We thank all involved persons for making their data available.

Tabelle 1: **Traditional Performance Measurement (1975–1994)**

Market Index				DAFOX			CDAX			DAX		
No.	Mutual Fund	Excess Rate of Return	Volatility	Alpha	Beta	Degree of Diversification	Alpha	Beta	Degree of Diversification	Alpha	Beta	Degree of Diversification
1	Adifonds	4.28 %	15.61 %	0.01 %	0.93	96.36 %	1.19 %	1.04	94.75 %	1.77 %*	0.85	92.68 %
2	Adiverba	3.68 %	14.39 %	0.07 %	0.79	81.28 %	1.08 %	0.88	79.24 %	1.64 %	0.70	72.75 %
3	Fondak	3.69 %	15.84 %	−0.66 %	0.95	97.31 %	0.53 %	1.06	95.64 %	1.15 %	0.86	91.92 %
4	Fondra	1.85 %	11.10 %	−1.16 %**	0.66	94.89 %	−0.33 %	0.74	93.24 %	0.08 %	0.60	90.77 %
5	Plusfonds	3.57 %	13.81 %	−0.16 %	0.81	93.91 %	0.85 %	0.92	93.85 %	1.38 %	0.74	90.22 %
6	Dekafonds	3.35 %	16.33 %	−1.12 %*	0.98	96.75 %	0.11 %	1.09	95.09 %	0.73 %	0.89	92.61 %
7	Concentra	4.83 %	15.60 %	0.58 %	0.93	95.96 %	1.72 %**	1.05	95.82 %	2.30 %**	0.86	94.59 %
8	DIT-Fonds	4.10 %	12.68 %	0.90 %	0.70	82.05 %	1.72 %	0.80	84.71 %	2.22 %*	0.64	78.58 %
9	Thesaurus	3.99 %	15.40 %	−0.20 %	0.92	95.81 %	0.93 %	1.03	95.62 %	1.52 %*	0.84	92.95 %
10	Investa	4.84 %	15.10 %	0.71 %	0.90	96.55 %	1.84 %**	1.01	95.25 %	2.39 %**	0.83	94.64 %
11	FT Frankfurter	5.19 %	12.69 %	1.79 %**	0.74	92.97 %	2.72 %**	0.84	91.96 %	3.17 %**	0.69	91.21 %
12	MK Alphakapital	3.03 %	13.79 %	−0.60 %	0.79	89.69 %	0.38 %	0.89	89.07 %	0.89 %	0.73	86.88 %
13	Oppenheim Privat	1.52 %	13.51 %	−1.90 %	0.75	82.64 %	−1.00 %	0.85	84.20 %	−0.51 %	0.69	80.75 %
14	SMH-Special I	4.54 %	13.74 %	1.00 %	0.78	86.11 %	1.95 %*	0.87	85.91 %	2.49 %*	0.70	80.31 %
15	Unifonds	4.15 %	14.96 %	0.05 %	0.90	96.91 %	1.18 %	1.00	95.19 %	1.72 %**	0.83	94.92 %
16	Main I-Universal	3.05 %	14.72 %	−0.88 %	0.86	92.07 %	0.16 %	0.97	92.71 %	0.74 %	0.78	88.23 %
17	Universal-Effekt	2.63 %	10.90 %	0.24 %	0.52	61.76 %	0.89 %	0.59	61.38 %	1.25 %	0.47	57.31 %
	Average	3.66 %	14.13 %	−0.08 %	0.82	90.18 %	0.94 %	0.92	89.63 %	1.47 %	0.75	86.55 %
	DAFOX	4.58 %	16.44 %									
	CDAX	2.96 %	14.56 %									
	DAX	2.94 %	17.64 %									

* Significance at the 10 % Level,
** Significance at the 5 % Level.

4.3 Exponential Performance Measurement

When using the EPM instead of Jensen's alpha we find nearly the same funds performance. As shown in table 2, the performance compared with Jensen's alpha decreases by a few basis points. At the same time, we achieve average betas slightly higher than those of the characteristic line. From this we can infer a negative timing component. The timing bias in Jensen's alpha for this sample is relatively small. But timing may still be a significant performance component. Only the comparison of the EPM with Jensen's alpha using proposition 3 allows to isolate the timing component.

A test for statistical significance of the estimated performance may proceed as follows. Under the hypothesis that there is neither selectivity nor timing, the expected performance is zero:

$$\alpha^{\mathrm{eu}}(r_P) = \mathrm{E}(w \cdot r_P) = \beta_P \cdot \mathrm{E}(w \cdot r_M) + \mathrm{E}(w \cdot \epsilon_P) = 0. \tag{26}$$

The variance of the estimator is

$$\begin{aligned} \sigma^2\big(\alpha^{\mathrm{eu}}(r_P)\big) &= \mathrm{E}(w \cdot \epsilon_P)^2 - \mathrm{E}^2(w \cdot \epsilon_P) = \mathrm{E}(w^2) \cdot \sigma^2(\epsilon_P); \qquad (27) \\ \mathrm{E}(w^2) &= \exp\left\{\frac{\mathrm{E}^2(r_M)}{\sigma_M^2}\right\} \\ \Rightarrow \quad \sigma^2\big(\widehat{\alpha}^{\mathrm{eu}}(r_P)\big) &= \exp\left\{\frac{\widehat{r}_M^2}{\widehat{\sigma}_M^2}\right\} \cdot \sigma^2(\epsilon_P). \end{aligned}$$

We assume that the estimator for the EPM is asymptotically normally distributed.[9] Therefore, in table 2 the significance of the observed performance is tested by a t-test. Due to the slightly lower performance and the higher standard errors compared to Jensen's alpha, the number of mutual funds with a statistically significant performance is lower.[10]

[9]Dybvig and Ross (1985) showed that portfolio rates of return may not be normally distributed if the portfolio manager possesses superior information even though the manager and the investor both face normal rates of return. This is due to the fact that portfolio rates of return are a product of portfolio choices and asset rates of return, both being functions of the normally distributed signal observed by the portfolio manager.

[10]For Jensen's alpha we have: $\sigma^2(\widehat{\alpha}_P) = \big(1 + \widehat{r}_M^2/\widehat{\sigma}_M^2\big) \cdot \sigma^2(\epsilon_P)$.

Tabelle 2: **Performance Ranking according to the EPM (1975–1994)**

Index	DAFOX			CDAX			DAX		
No.	Performance	Average Beta	Rank	Performance	Average Beta	Rank	Performance	Average Beta	Rank
1	−0.09 %	0.95	9	1.14 %	1.06	9	1.75 %	0.86	9
2	−0.04 %	0.81	8	1.02 %	0.90	7	1.61 %	0.71	7
3	−0.77 %	0.97	12	0.48 %	1.08	12	1.13 %	0.87	12
4	−1.24 %	0.67	16	−0.37 %	0.75	16	0.06 %	0.61	16
5	−0.25 %	0.83	10	0.80 %	0.93	10	1.36 %	0.75	10
6	−1.22 %	1.00	15	0.06 %	1.11	15	0.71 %	0.90	15
7	0.48 %	0.95	5	1.68 %*	1.06	6	2.28 %**	0.87	5
8	0.82 %	0.72	3	1.68 %	0.82	2	2.20 %	0.64	3
9	−0.30 %	0.94	11	0.88 %	1.05	11	1.49 %	0.85	11
10	0.62 %	0.92	4	1.80 %*	1.03	5	2.37 %**	0.84	4
11	1.71 %*	0.76	1	2.68 %**	0.85	1	3.15 %**	0.69	1
12	−0.69 %	0.81	13	0.34 %	0.91	13	0.87 %	0.74	13
13	−2.02 %	0.77	17	−1.06 %	0.87	17	−0.54 %	0.70	17
14	0.95 %	0.79	2	1.93 %	0.88	4	2.48 %	0.70	2
15	−0.04 %	0.92	7	1.14 %	1.02	8	1.70 %*	0.83	8
16	−0.99 %	0.88	14	0.11 %	0.99	14	0.72 %	0.79	14
17	0.16 %	0.54	6	0.85 %	0.60	3	1.23 %	0.47	6
Average	−0.17 %	0.84		0.89 %	0.94		1.45 %	0.75	

* Significance at the 10 % Level,
** Significance at the 5 % Level.

Roll's (1978) critique of the CAPM focuses on the ambiguity of performance measurement that results from an inefficient market proxy. The EPM is based on a local market model. Therefore, this critique does not apply here. Nonetheless, the local market model assumes a locally efficient market proxy. Therefore, performance measurement depends on the employed market proxy. Performance rankings according to Treynor's ratio with *average* beta provide the following picture: Performance measurement with the DAFOX and CDAX leads to a nearly identical performance ranking. However, using the DAX causes changes in the ranking. Nonetheless, the Spearman rank correlation coefficients are very high. All probabilities of falsely rejecting the null-hypothesis that the rankings are uncorrelated are below 1 %.

Clearly, the estimated performance depends on the benchmark used. The average performance increases by 106 basis points and 162 basis points per year using the CDAX and the DAX, respectively, instead of the DAFOX. But figures 1 to 3 visualize the insensitivity of the mutual funds's ranking with respect to the market proxies chosen.

Figure 1: **Treynor's Ratio with Average Beta (1975–1994)**

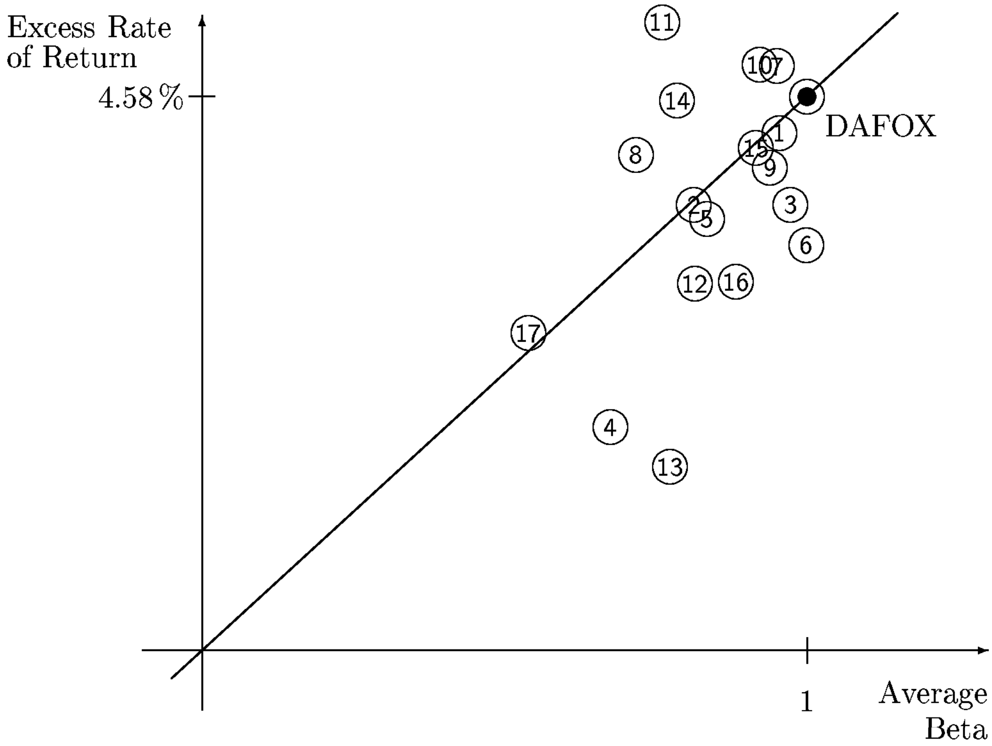


Figure 2: **Treynor's Ratio with Average Beta (1975–1994)**

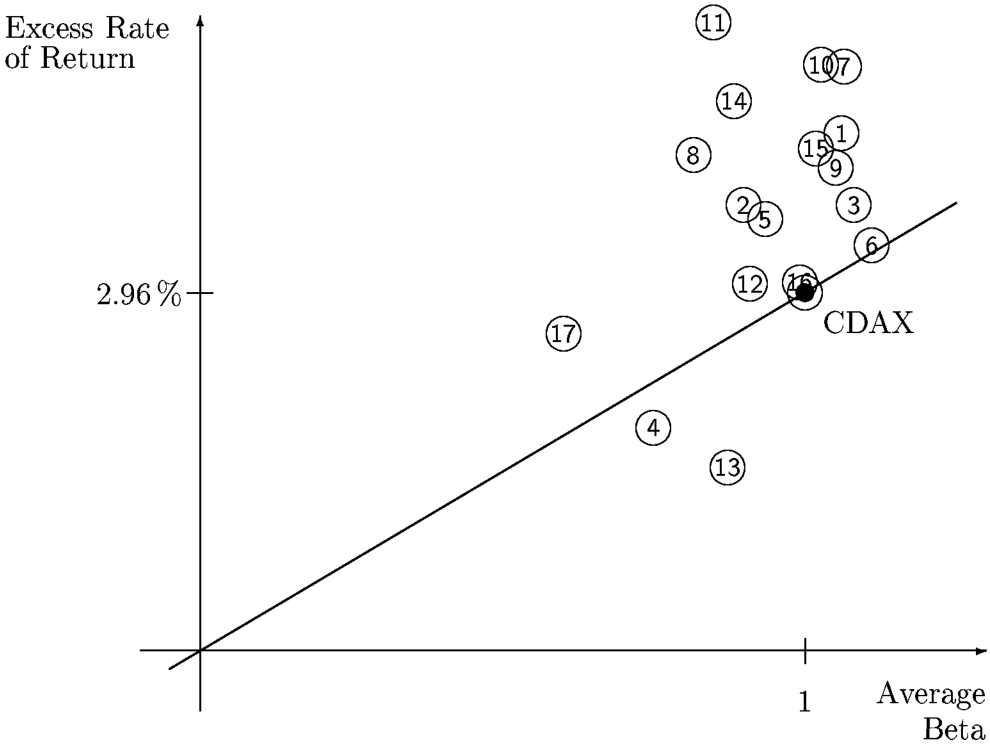
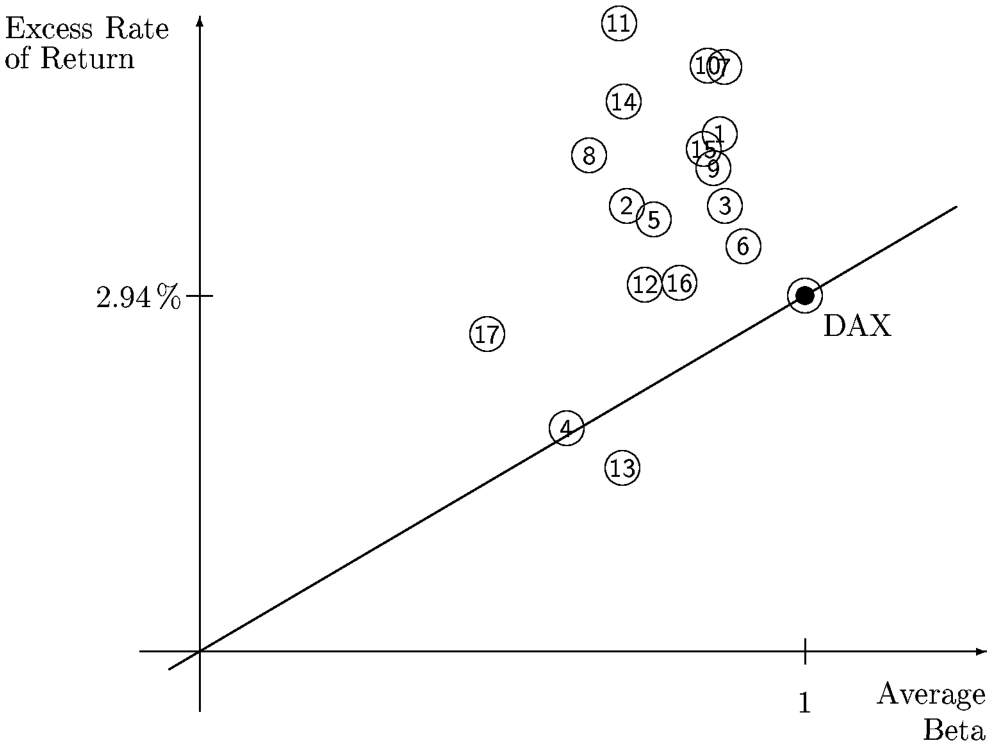


Figure 3: **Treynor's Ratio with Average Beta (1975–1994)**



4.4 Performance Attribution

Performance attribution decomposes mutual fund performance in a timing component and a selectivity component. As mentioned above, we know from the comparison of tables 1 and 2 that the lower performance estimates according to the EPM (compared with Jensen's alpha) express negative timing components. This is compensated by positive selectivity except for two funds when using the DAFOX. Table 3 shows that this result is independent from the market proxy chosen.

Table 3: **Performance Attribution (1975–1994)**

Index	DAFOX		CDAX		DAX	
No.	Selectivity	Timing	Selectivity	Timing	Selectivity	Timing
1	1.18 %	−1.27 %	2.29 %	−1.15 %	2.52 %	−0.77 %
2	1.35 %	−1.39 %	2.29 %	−1.27 %	2.58 %	−0.98 %
3	0.56 %	−1.32 %	1.67 %	−1.19 %	1.94 %	−0.81 %
4	−0.27 %	−0.97 %	0.55 %	−0.92 %	0.69 %	−0.62 %
5	0.93 %	−1.18 %	1.87 %	−1.07 %	2.08 %	−0.72 %
6	0.01 %	−1.23 %	1.18 %	−1.12 %	1.43 %	−0.72 %
7	1.70 %	−1.21 %	2.79 %	−1.11 %	2.98 %	−0.70 %
8	1.90 %	−1.08 %	2.67 %	−0.99 %	2.88 %	−0.68 %
9	0.95 %	−1.25 %	2.03 %	−1.15 %	2.26 %	−0.76 %
10	1.74 %	−1.11 %	2.80 %	−1.00 %	3.00 %	−0.63 %
11	2.65 %	−0.94 %	3.57 %	−0.90 %	3.71 %	−0.55 %
12	0.47 %	−1.16 %	1.40 %	−1.06 %	1.61 %	−0.74 %
13	−0.47 %	−1.55 %	0.38 %	−1.45 %	0.51 %	−1.04 %
14	1.52 %	−0.57 %	2.48 %	−0.55 %	2.75 %	−0.26 %
15	1.14 %	−1.17 %	2.23 %	−1.09 %	2.41 %	−0.71 %
16	0.36 %	−1.35 %	1.34 %	−1.23 %	1.59 %	−0.87 %
17	1.22 %	−1.07 %	1.83 %	−0.98 %	1.95 %	−0.72 %
Average	1.00 %	−1.17 %	1.96 %	−1.07 %	2.17 %	−0.72 %

With respect to the DAFOX, we find an average performance of 1.00 % per year resulting from stock picking while poor timing overcompensated selectivity. The results are different using another proxy. The average selectivity increases and the negative timing component decreases when the whole analysis is based on the CDAX. The same result holds when using the DAX as market proxy. The index sensitivity becomes more evident when looking at figures 4 to 6. The relative positions of the mutual funds do not change very much.

Figure 4: **Timing and Selectivity (1975–1994)**

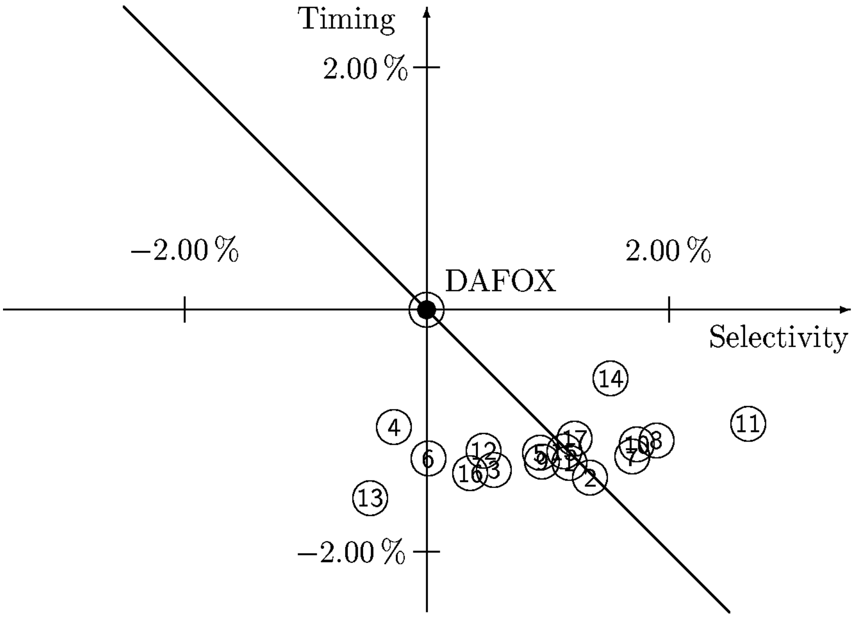


Figure 5: **Timing and Selectivity (1975–1994)**

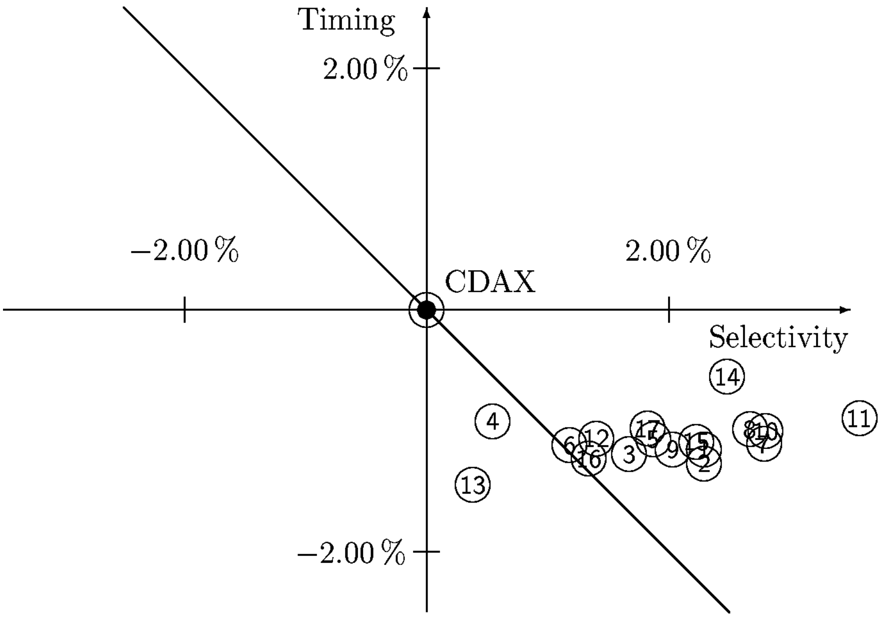
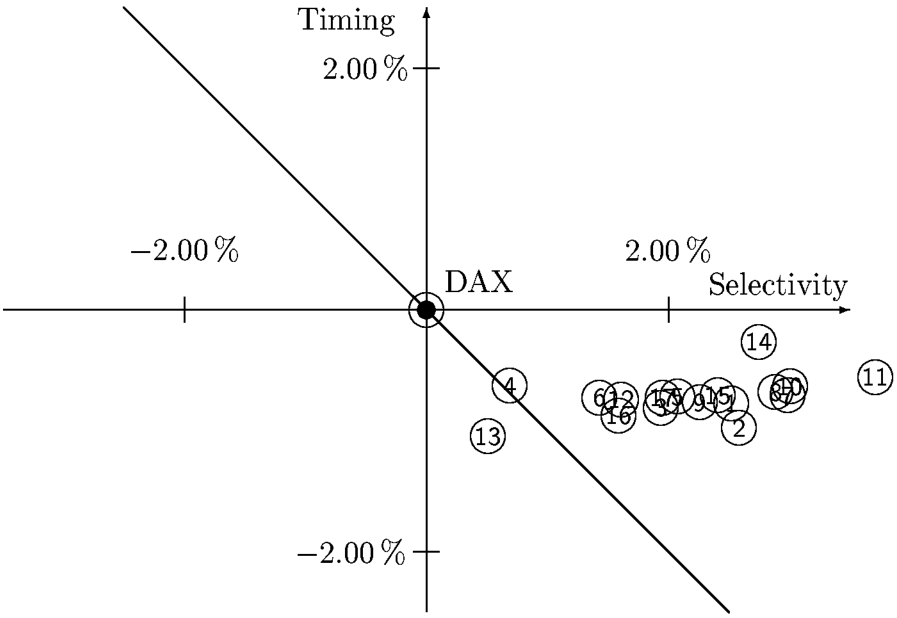


Figure 6: **Timing and Selectivity (1975–1994)**



4.5 Persistence of Performance

Finally, we look at the persistence of performance. We split the 20-year-sample period into two subperiods: from 1975 to 1984 and from 1985 to 1994. Table 4 presents the results.

In the first period from 1975 to 1984 the mutual funds achieved a small but positive timing component. Simultaneously, the majority of the mutual funds showed a negative selectivity that in most cases causes a negative total performance. However, this does not apply to the second period. With six exceptions the mutual funds showed positive selectivity but all of them obtained negative timing components. It is conceivable that the crashes in 1987 and 1990 are responsible for the negative timing components in the second subperiod. Careful transactions might have triggered that the mutual funds did not participate at increasing prices to the peak.

Tabelle 4: **Performance (1975–1984 and 1985–1994)**

Period	1975–1984					1985–1994				
No.	Performance	Average Beta	Rank	Selectivity	Timing	Performance	Average Beta	Rank	Selectivity	Timing
1	−0.15 %	0.94	8	−0.63 %	0.48 %	−0.05 %	0.94	9	1.25 %	−1.30 %
2	2.49 %*	0.75	1	2.22 %	0.27 %	−3.26 %*	0.81	17	−1.88 %	−1.37 %
3	−0.14 %	0.93	7	−0.66 %	0.52 %	−1.51 %*	0.96	13	−0.16 %	−1.35 %
4	−0.32 %	0.66	10	−0.64 %	0.33 %	−2.15 %**	0.66	16	−1.16 %	−0.98 %
5	−0.78 %	0.73	13	−1.10 %	0.32 %	0.78 %	0.86	7	1.98 %	−1.21 %
6	−1.72 %**	0.92	15	−2.25 %	0.53 %	−0.46 %	1.01	10	0.78 %	−1.25 %
7	−0.51 %	0.89	11	−1.00 %	0.49 %	1.99 %*	0.96	5	3.25 %	−1.26 %
8	−0.57 %	0.54	12	−0.71 %	0.14 %	3.00 %*	0.77	2	4.10 %	−1.09 %
9	−0.95 %	0.85	14	−1.41 %	0.46 %	0.97 %	0.95	6	2.27 %	−1.30 %
10	−0.35 %	0.86	9	−0.85 %	0.50 %	1.99 %**	0.93	4	3.13 %	−1.15 %
11	1.12 %	0.78	3	0.77 %	0.35 %	2.36 %**	0.74	3	3.32 %	−0.96 %
12	0.25 %	0.75	4	0.00 %	0.26 %	−1.28 %	0.82	12	−0.14 %	−1.14 %
13	−1.78 %	0.69	17	−1.93 %	0.15 %	−1.93 %	0.78	15	−0.27 %	−1.66 %
14	−1.47 %	0.73	16	−1.99 %	0.52 %	3.71 %**	0.79	1	4.26 %	−0.55 %
15	−0.04 %	0.92	6	−0.66 %	0.61 %	0.14 %	0.89	8	1.39 %	−1.25 %
16	−0.02 %	0.67	5	−0.38 %	0.36 %	−1.21 %	0.94	11	0.16 %	−1.37 %
17	1.47 %	0.57	2	1.33 %	0.14 %	−1.25 %	0.51	14	−0.14 %	−1.11 %
Average	−0.20 %	0.78		−0.58 %	0.38 %	0.11 %	0.84		1.30 %	−1.19 %

* Significance at the 10 % Level,
** Significance at the 5 % Level.

Figures 7 and 8 show the components of timing and selectivity in mutual fund performance for both subperiods.

Figure 7: **Timing and Selectivity (1975–1984)**

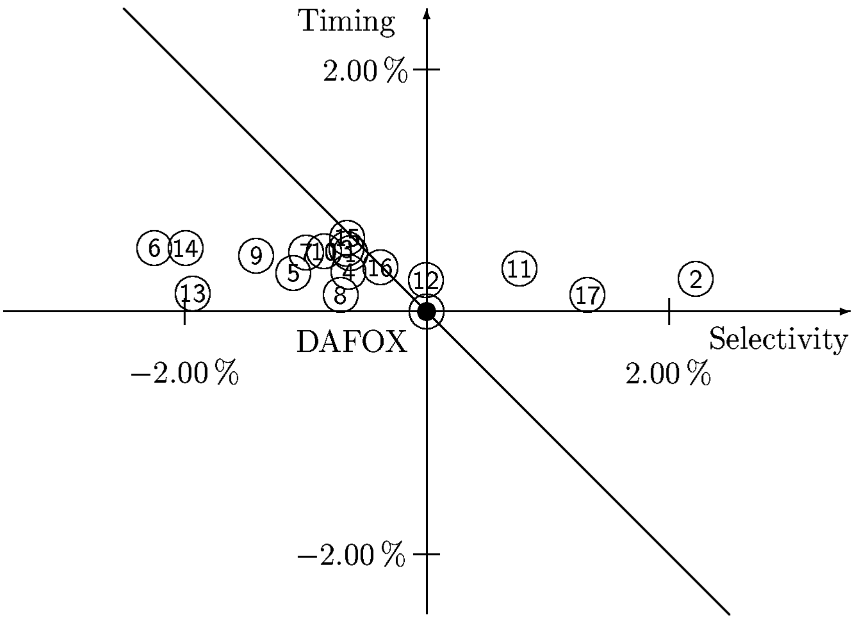
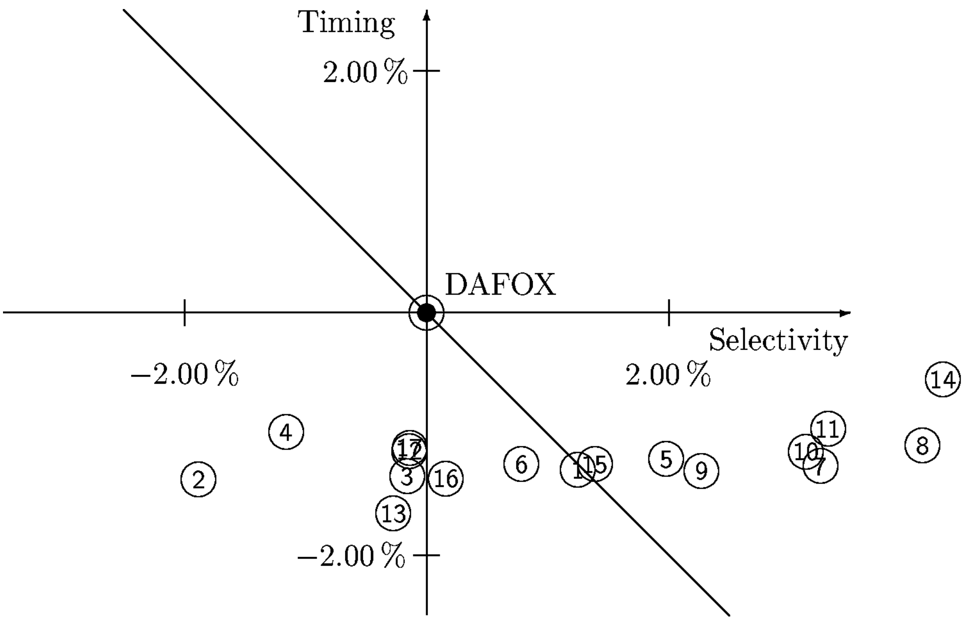


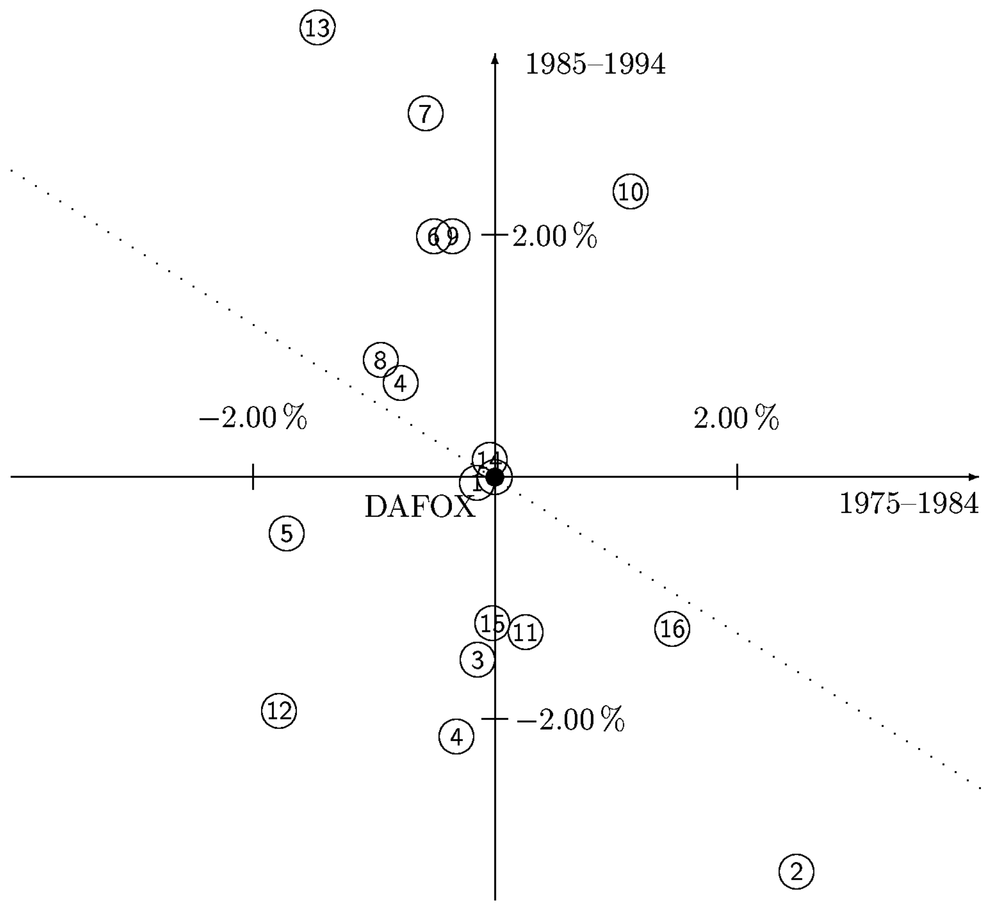
Figure 8: **Timing and Selectivity (1985–1994)**



Persistence of performance is important for an investor who decides to invest in a mutual fund because of its past performance. Usually, two procedures are used to test whether a mutual fund with positive or negative performance, respectively, in the first period belongs to the same class in the second period:

1) The ranking can be used for a rank correlation test. Therefore, Treynor's ratio with average beta has to be computed. Using the data shown in table 4 the Spearman rank correlation coefficient is −0.3382. At a confidence level of 10 % the hypothesis that the rankings are uncorrelated cannot be refused.

Figure 9: **Performance (1975–1984 and 1985–1994)**



2) Regression of the performance of the second period on the performance of the first period shows a negative correlation. The slope of the dotted regression line shown

in figure 9 is not different from zero at a confidence level of 10 %. Therefore, we cannot claim that previous performance affects future performance.

Our sample does not show persistence of performance.[11] The mutual fund with the highest performance in the first subperiod (no. 2) obtained the lowest performance in the second subperiod whereas the mutual fund with the second lowest performance in the first period (no. 14) achieved the best performance in the second period.

5 External Versus Internal Performance Attribution

For positive period weighting measures within the local market model, according to assumption (A1) a benchmark has to be provided that is an efficient combination of assets $i = 1, \ldots, N$. Therefore, the following return generating process is assumed:

$$r_{it} \;=\; \beta_i \cdot r_{Mt} + \epsilon_{it}; \quad i = 1, \ldots, N; \quad t = 1, \ldots, T.$$

Let x_{it} denote the fraction of the portfolio invested in asset i at time t. The total period portfolio excess rate of return is

$$\widehat{r}_P \;=\; \frac{1}{T} \cdot \sum_{t=1}^{T} r_{pt} = \frac{1}{T} \cdot \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot r_{it} = \frac{1}{T} \cdot \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot (\beta_i \cdot r_{Mt} + \epsilon_{it}). \quad (28)$$

Knowing the composition of the portfolio allows to compute the respective portfolio beta and portfolio residual. By that it is possible to quantify the portfolio manager's ability of timing and selectivity. At time t, we get for the beta and the residual:

$$\beta_{Pt} = \sum_{i=1}^{N} x_{it} \cdot \beta_i \quad \text{und} \quad \epsilon_{Pt} = \sum_{i=1}^{N} x_{it} \cdot \epsilon_{it}. \quad (29)$$

[11] Our result is in accordance with empirical results based on traditional performance measures. Grinblatt and Titman (1992) find no persistence in performance of US mutual funds. Wittrock (1995, pp. 451–460) finds no persistence for German mutual funds. Recent studies find little persistence in the ranking of mutual fund performance for US mutual funds, see, for example, Gruber (1996).

We stress the interdependence of external and internal performance measurement with the following

PROPOSITION 5: *Assume that the local market model holds. Then the performance consisting of timing and selectivity equals the sum of covariances between portfolio weight and rate of return, i.e. the Grinblatt and Titman (1993) internal performance measure:*

$$\underbrace{\widehat{\text{Cov}}(\beta_P, r_M)}_{\textit{timing}} + \underbrace{\widehat{\epsilon}_P}_{\textit{selectivity}} = \underbrace{\sum_{i=1}^{N} \widehat{\text{Cov}}(x_i, r_i)}_{\textit{internal performance measure}} \quad . \tag{30}$$

Furthermore, the Grinblatt and Titman (1993) internal performance measure uses a passive strategy with average portfolio weights as a benchmark:

$$\underbrace{\sum_{i=1}^{N} \widehat{\text{Cov}}(x_i, r_i)}_{\textit{internal performance measure}} = \widehat{r}_P - \underbrace{\widehat{\beta}_P \cdot \widehat{r}_M}_{\textit{benchmark return}} \quad . \tag{31}$$

PROOF:

$$\begin{aligned}
\widehat{r}_P - \widehat{\beta}_P \cdot \widehat{r}_M &= \widehat{\text{Cov}}(\beta_P, r_M) + \widehat{\epsilon}_P \\
&= \frac{1}{T} \cdot \sum_{t=1}^{T} \left((\beta_{Pt} - \widehat{\beta}_P) \cdot r_{Mt} + \epsilon_{Pt} \right) \\
&= \frac{1}{T} \cdot \sum_{t=1}^{T} \left(\sum_{i=1}^{N} x_{it} \cdot (\beta_i \cdot r_{Mt} + \epsilon_{it}) \right) - \widehat{\beta}_P \cdot \widehat{r}_M \\
&= \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot r_{it} - \sum_{i=1}^{N} \widehat{x}_i \cdot \underbrace{\beta_j \cdot \widehat{r}_M}_{\widehat{r}_i} \\
&= \sum_{i=1}^{N} \widehat{\text{Cov}}(x_i, r_i).
\end{aligned}$$

□

The sum of covariances between portfolio fractions of single assets and their excess rates of return represents the *internal performance measure* of Grinblatt and Titman (1993). This sum of covariances can be used as a performance measure since it assigns zero performance to a passive strategy with unchanged portfolio weights.[12] This performance measure is quite intuitive. If the portfolio manager correctly anticipates price movements, the change of portfolio weights is directly related to price changes. This gives a positive covariance of portfolio weight and rate of return of the corresponding asset.

An inefficient market proxy leads to deviations from the local market model, i.e. an alpha different from zero. The inefficient market proxy is denoted by I. We assume a one-factor model as return generating process for asset excess rates of return:

$$r_{it} \;=\; \alpha_i + \beta_i \cdot r_{It} + \epsilon_{it}; \quad i = 1, \ldots, N; \quad t = 1, \ldots, T. \tag{32}$$

With $\widehat{\alpha}_P \equiv \sum_{i=1}^{N} \widehat{x}_i \cdot \alpha_i$ this gives the following rate of return decomposition:

$$\widehat{r}_P \;=\; \underbrace{\widehat{\beta}_P \cdot \widehat{r}_I}_{\text{benchmark return}} + \underbrace{\widehat{\alpha}_P}_{\text{allocation}} + \underbrace{\underbrace{\widehat{\mathrm{Cov}}(\beta_P, r_I)}_{\text{timing}} + \underbrace{\widehat{\epsilon}_P}_{\text{selectivity}}}_{\sum_{i=1}^{N} \widehat{\mathrm{Cov}}(x_i, r_i)} . \tag{33}$$

Thus, the internal performance measure is not able to cover the component of the portfolio excess rate of return resulting from the alphas. Moreover, it measures the sum of timing component related to the inefficient benchmark and selectivity which summarizes above average and below average rates of return of single assets. In addition to the result of Heinkel and Stoughton (1997) this emphasizes that the internal performance measure depends on a benchmark.

The allocation component results from the average portfolio fractions. It gives the component of the portfolio excess rate of return by which the average strategy

[12]The covariance $\mathrm{Cov}(x_i, r_i) = \mathrm{Cov}\big(x_i - \mathrm{E}(x_i), r_i\big)$ represents the *continuous performance measure* of Heinkel and Stoughton (1997). It compares deviations from the average portfolio fraction of a asset with its respective rates of return. However, the internal performance measure of Grinblatt and Titman (1993) uses the changes of the portfolio fraction: $\mathrm{Cov}(x_{it} - x_{i,t-1}, r_{it})$. Both performance measures are identical if current rate of return and portfolio fraction of the last period are uncorrelated. The similar performance measure of Cornell (1979) does not focus on changes of the portfolio fraction but on above average or below average rates of return.

exceeds the passive strategy. This component of the portfolio rate of return belongs to the portfolio manager's ability of diversification given an inefficient benchmark. Thus, the allocation component is part of the performance.

This fundamental aspect remains true even when assuming a linear multi-factor model which can be justified taking an arbitrage free market into consideration. Here, the timing component corresponds to the covariances of factor sensitivities and systematic risk factors. The representation of selectivity and allocation components does not change. This means that internal performance measurement for this class of return generating processes does not consider the allocation component whereas the EPM does.

6 Summary

Traditional performance measurement is mainly criticized on two grounds: Firstly, an inefficient benchmark may lead to mismeasurement of mutual fund performance as single assets even in capital market equilibrium may show positive as well as negative alphas. In this situation, it is not clear whether the performance is caused by portfolio management or by inefficiency of the benchmark. Secondly, performance measurement based on the characteristic line requires a constant portfolio beta. Rearranging the portfolio composition on timing signals causes changes in the portfolio beta which subsequently lead to a bias in performance measurement.

Members of the PAPM-class overcome these weaknesses but usually have the disadvantage of a timing bias. The EPM proposed in this paper avoids this disadvantage. We can completely eliminate the timing bias with the EPM. The comparison with Jensen's alpha enables us to isolate timing and selectivity. By adjusting with the average portfolio beta a performance ratio can be computed that allows a consistent ranking of mutual fund performance. This ratio is independent from the portfolio manager's risk tolerance.

Our empirical study on German mutual fund performance using the EPM leads to the following main results: Firstly, index sensitivity affects performance measurement. We have found that the broader the market proxy the lower is the performance. However, the performance ranking is not very sensitive with respect to market proxy choice. Secondly, performance attribution showed positive selectivity and negative timing for the period from 1975 to 1994. But the results depend on the analyzed time period. Previous performance does not allow a forecast.

Finally, within the local market model we showed that the EPM and external performance attribution, respectively, give the same information on the portfolio manager's abilities as internal performance measurement. The latter does not only rely on return data but also on information on portfolio weights.

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