Sequential Warrant Exercise in Large Trader Economies

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Motivation

- Not only hedge funds focusing on convertible arbitrage hold often substantial parts of convertible issues.
- Such investors act very often as non-pricetaker and must be deemed as "large trader".
- Classical literature on valuation and exercising of convertibles considers only a simplified capital structure without (straight) dept of the issuing firm.

Related literature and its key message

- **Emanuel (1983), Constantinides (1984), and others:**
  - *"... warrant valuation and exercise strategy differ fundamentally from call option valuation - sequential exercise is benefical to "large" warrantholders."*
- **Spatt and Sterbenz (1988):**
  - *"Sequential exercise may be advantageous for monopoly and oligopoly warrantholders, but there are reinvestment policies for which sequential exercise is not advantageous."*
- **Bühler and Koziol (2002):**
  - *"Partial exercise can be optimal for pricetakers in the presence of additional debt."*

Main findings

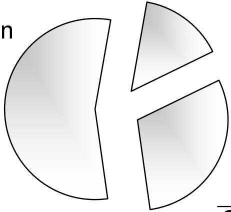
- We present sufficient conditions for the non-optimality of sequential exercise of American-type warrants.
- For a *realistic* parameter setting it turns out that exercising warrants sequentially is *not* beneficial to non-pricetaking ("large") warrantholders.
- This result, however, does not justify in general the simplifiying restriction that warrants or convertible securities are valued as if exercised as a block.

Model

Capital structure of the firm

$(n - m_t)$ outstanding warrants with total value $(n - m_t)\ W_t(V_t)$, maturity $T < T_D$, and strike price K

debt $D_t(V_t)$ (zero coupon bond with face value F and maturity T_D)



$(N + m_t)$ shares of common stock with total value $\overline{S}_t(V_t) = (N + m_t)\ S_t(V_t)$

with firm value

$$V_t = (N + m_t)\ S_t + (n - m_t)\ W_t + D_t \text{ for all } t \in [0, T)$$

Further assumptions

- Exercise proceeds are used to rescale the firm's investment. At exercise times t_k the firm value jumps to

$$V_{t_k} \quad = \quad A_{t_k} + \sum_{j=1}^{k} m'_{t_j} K \frac{A_{t_k}}{A_{t_j}},$$

  where A_t denotes the price of the "average" asset in which the firm invests, and m'_t denotes the number of warrants exercised at time t.

- No dividend payments.

- Warrantholder do not hold shares of common stock.

Definitions

D1 Warrantholders follow a so-called *sequential exercise* strategy if they exercise American-type warrants before maturity. Otherwise the warrantholders follow a so-called *block exercise* strategy if the number of warrants exercised at the maturity date is given by

$$m_T = \begin{cases} 0 & \text{for} \quad \frac{1}{N+n}\,\overline{S}_T\left(V_T\right) \in [0, K) \\ n & \text{for} \quad \frac{1}{N+n}\,\overline{S}_T\left(V_T\right) \in [K, \infty), \end{cases}$$

or they follow a so-called *partial exercise* strategy.

D2 The *partial exercise option* is the option to follow a partial exercise strategy instead of a block exercise strategy. The *sequential exercise option* is the option to follow a sequential exercise strategy instead of a partial exercise strategy.

Non–cooperative, non-zero-sum game

- We model the warrantholders' exercise behavior as a noncooperative game and consider a Nash equilibrium as an optimal exercise strategy for the warrantholders.
- While Constantinides (1984) and other authors analyse a zero-sum game between the warrantholders and the stockholders (as passive players),
- our game is not zero-sum: there is a wealth transfer from the stockholders and the warrantholders to the debtholders when warrants are exercised (like in Bühler and Koziol (2002), Koziol (2003, 2006), and Kapadia and Willette (2005)).

Payoff function before maturity

- for a pricetaking warrantholder p

$$\pi_t^p \left(m^p, m, V_t\right) = m_t^p \left(S_t(V_t) - K\right) + \left(n^p - m_t^p\right) \, W_t \, \left(V_t\right),$$

- for a non-pricetaker:

$$\pi_t^L \left(m^L, m^{-L}, V_t\right) = m_t^L \left(S_t(V_t) - K\right) + \left(n^L - m_t^L\right) \, W_t \, \left(V_t\right).$$

His exercise policy influences the firm value and in particular the stock price $S_t(V_t)$.

Payoff function at maturity

- is *linear* (in the number of warrants exercised by himself) for a pricetaking warrantholder:

$$\pi_T^p(m^p, m, V_T) = m_T^p \left(\frac{\overline{S}_T(V_T)}{N + m_T} - K \right) ,$$

- is *quasi-concave* (with respect to m_T^L, see Linder/Trautmann, 2006) for a non-pricetaking warrantholder:

$$\pi_T^L(m^L, m^{-L}, V_T) = m_T^L \left(\frac{\overline{S}_T(V_T)}{N + m_T^L + m_T^{-L}} - K \right) .$$

Exercise strategies in a Nash equilibrium

Extending the results of Koziol (2006), Kapadia and Willette (2005), and Linder and Trautmann (2006) we can show that the following strategy is a Nash equilibrium:

$$(m_T^{p\,*}, m_T^{L_1\,*}, m_T^{L_2\,*}, \dots, m_T^{L_Z\,*}) = \begin{cases} (0, 0, 0, \dots, 0) & \text{for } V_{T^-} \in [0, \underline{V}) \\ (x^*, 0, 0, \dots, 0) & \text{for } V_{T^-} \in [\underline{V}, \underline{V}_1) \\ (n^p, x_1^*, x_1^*, \dots, x_1^*) & \text{for } V_{T^-} \in [\underline{V}_1, \underline{V}_2) \\ (n^p, n^{L_1}, x_2^*, \dots, x_2^*) & \text{for } V_{T^-} \in [\underline{V}_2, \underline{V}_3) \\ \vdots & \\ (n^p, n^{L_1}, \dots, n^{L_{Z-1}}, x_Z^*) & \text{for } V_{T^-} \in [\underline{V}_Z, \overline{V}_Z) \\ (n_L, n^{L_1}, n^{L_2}, \dots, n^{L_Z}) & \text{for } V_{T^-} \in [\overline{V}_Z, \infty) \end{cases}$$

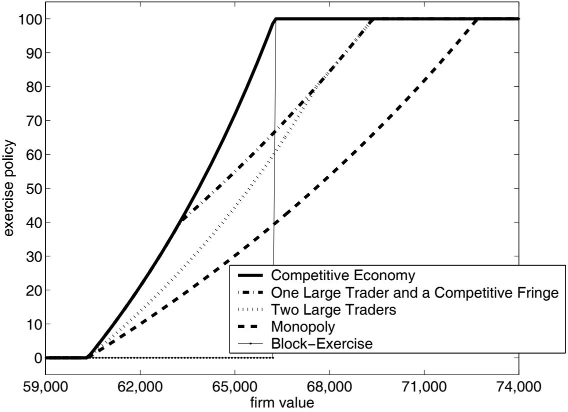
where the critical firm values $\underline{V}$, $\underline{V}_1$, $\underline{V}_2$, $\underline{V}_3$, $\dots$, $\underline{V}_Z$ and $\overline{V}_Z$ solve

$$
\begin{aligned}
\frac{1}{N}\,\overline{S}_T(\underline{V}) &= K \\
\frac{1}{N+n^p}\,\overline{S}_T(\underline{V}_1 + n^p K) &= K \\
\frac{\partial}{\partial m_T^{L_1}}\,\pi_T^{L_1}\left(n^{L_1}, n^p + (Z-1)n^{L_1}, \underline{V}_2 + m_T K\right) &= 0 \\
\frac{\partial}{\partial m_T^{L_2}}\,\pi_T^{L_2}\left(n^{L_2}, n^p + n^{L_1} + (Z-2)n^{L_2}, \underline{V}_3 + m_T K\right) &= 0 \\
&\vdots \\
\frac{\partial}{\partial m_T^{L_{Z-1}}}\,\pi_T^{L_{Z-1}}\left(n^{L_{Z-1}}, n^p + n^{L_1} + \ldots + n^{L_{Z-2}} + n^{L_{Z-1}}, \underline{V}_Z + m_T K\right) &= 0 \\
\frac{\partial}{\partial m_T^{L_Z}}\,\pi_T^{L_Z}\left(n^{L_Z}, n^p + n^{L_1} + \ldots + n^{L_{Z-1}}, \overline{V}_Z + n\ K\right) &= 0
\end{aligned}
$$

And the exercise policies $x^*, x_1^*, x_2^*, \ldots, x_Z^*$ are the solutions of

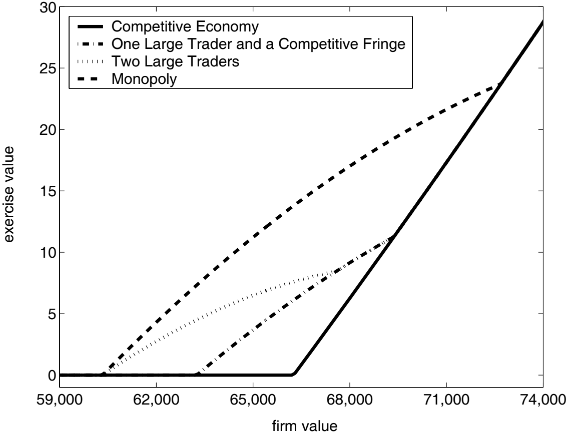
$$
\begin{aligned}
\frac{1}{N + x^*}\ \overline{S}_T\Big(V_{T^-} + x^* K\Big) &= K \\
\frac{\partial}{\partial m_T^{L_1}}\ \pi_T^{L_1}(x_1^*, n^p + (Z-1)\ x_1^*, V_{T^-} + m_T K) &= 0 \\
\frac{\partial}{\partial m_T^{L_2}}\ \pi_T^{L_2}(x_2^*, n^p + n^{L_1} + (Z-2)\ x_2^*, V_{T^-} + m_T K) &= 0 \\
&\vdots \\
\frac{\partial}{\partial m_T^{L_Z}}\ \pi_T^{L_Z}\Big(x_Z^*, n^p + n^{L_1} + \ldots + n^{L_{Z-1}}, V_{T^-} + n\ K\Big) &= 0
\end{aligned}
$$

Optimal exercise policies of European-type warrants



Parameters:
$r = 5\%$,
$\sigma = 0.25$,
$F = 80,000$,
$T_D - T = 4$,
$N = 100$,
$n = 100$,
$n^{-L} = n^b = 40$,
$K = 100$

Exercise values of European-type warrants



Parameters:

$r = 5\%$,

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$F = 80,000$,

$T_D - T = 4$,

$N = 100$,

$n = 100$,

$n^{-L} = n^b = 40$,

$K = 100$

Sequential exercise of American-type warrants

- Emanuel (1983), Constantinides (1984), and others emphasize the *potential* advantage of sequential exercise strategies by "large" warrantholders, even absent regular dividend payments. The following example illustrates this advantage.

- We assume that the firm's assets follows a binomial process with two periods starting in $t = 0$ and $t = T$. In each period the firm's asset can increase by 27% or decrease by 25% (the interest rate equals $r = 1\%$ then the risk neutral probability for an increase is $q = 0.5$).

Beneficial sequential exercise: an example

With $N = n = K = 100$, and $V_0 = A_0$ we get

$$V_0 = 160,000 + 100 m_0$$

$$V_T^u = 203,200 + 27 m_0 + 100 m_T$$

$$V_{T_D}^{uu} = 148,064 + 34.29 m_0 + 127 m_T$$

$$V_{T_D}^{ud} = 42,400 + 20.25 m_0 + 75 m_T$$

$$V_T^d = 120,000 - 25 m_0 + 100 m_T$$

$$V_{T_D}^{du} = 42,400 - 31.75 m_0 + 127 m_T$$

$$V_{T_D}^{dd} = 0$$

Stock price, warrant price and the debt value satisfy

$$
\begin{aligned}
S_0(V_0) &= \frac{1}{1+r}\left(qS_T(V_T^u) + (1-q)S_T(V_T^d)\right) = \frac{1}{(1+r)^2}\left(332.21 + 0.03m_0\right) \\
W_0(V_0) &= S_0(V_0) - \frac{1}{1+r}100 \\
D_0(V_0) &= \frac{1}{(1+r)^2}\left(106,875 - 4.69m_0\right) .
\end{aligned}
$$

- Pricetaking warrantholders are better off not to exercise warrants since $S_0(A_0) - K < W_0(A_0)$.
- Non-pricetaker L will exercise either all warrants or no warrant at all since

$$
\frac{\partial}{\partial m_0^L}\pi_0^L(m^L, 0, V_0) = \left(\frac{1}{1+r}100 - 100\right) + n^L\frac{0.03}{(1+r)^2}.
$$

- The requirement $\partial\pi_0^L(m^L, 0, V_0)/\partial m_0^L > 0$ is equivalent to $n^L > 33,67$. That is, if warrantholder L owns more than 33.67 warrants he exercises all his warrants, otherwise none.

- Price impacts for different market regimes:

	Competitive economy	One large trader ($n^L = 33$)	One large trader ($n^L = 66$)	Monopoly
S_o	325.66	325.66	327.61	328,61
W_o	226.65	226.65	228.60	229,60
D_o	104,769.14	104,769.14	104,465.70	104,309.38

Sufficient conditions for no sequential exercise

Condition I (model-independent)

Warrantholder L's sequential exercise option has zero value if the following upper bound on the wealth transfer per warrant from stock- and bondholders to warrantholder L is less than the present value of earnings from investing K dollars for T periods:

$$K\left(1-e^{-rT}\right) > K\frac{n^L}{N+n^L}.$$

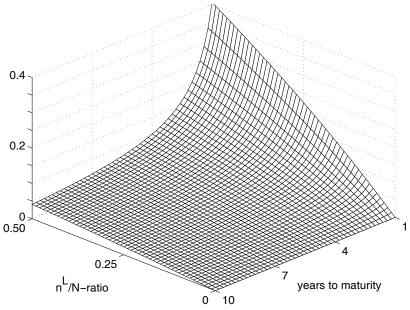
Condition II (model-dependent)

Warrantholder L's sequential exercise option has zero value if

$$K\left(1-e^{-rT}\right) > K\frac{n^L}{N+n^L}\left(\frac{C_0(V_0,V_0)}{V_0}\right).$$

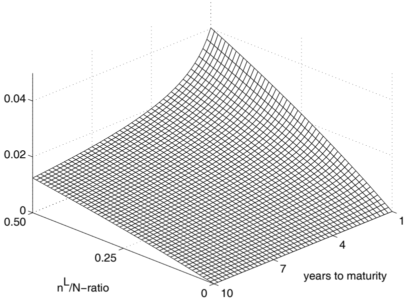
Lower bounds on interest rate levels preventing sequential exercise according to

Condition I



(model – independent)

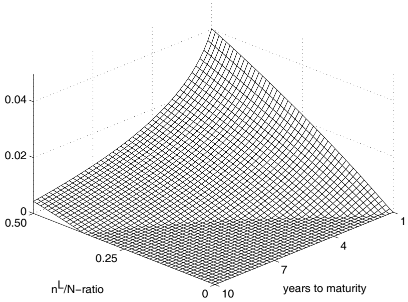
Condition II



(model – dependent)

Lower bounds on interest rate levels preventing sequential exercise according to

Condition III



$$\begin{aligned}(\text{Asset value } A_0 &= 40.000,\\ \text{Face value } F &= 10.000.)\end{aligned}$$

Condition III

$$\begin{aligned}(\text{Asset value } A_0 &= 40.000,\\ \text{Face value } F &= 15.000.)\end{aligned}$$

Conclusion

- This paper clarifies under which conditions sequential exercise of American-type warrants is beneficial to warrantholders.
- We present three different (sufficient) conditions for the non-optimality of sequential exercise.
- The advantage of sequential exercise of warrants decreases with increasing interest rates, increasing time to maturity, and decreasing ownership concentration.
- These results, however, do not justify in general the simplifying restriction that warrants are valued as if exercised as a block.
- The partial exercise option has namely a positive value if (and only if) the firm has debt in its capital structure and there is at least one non-price taking warrantholder.