A Large Trader's Impact on Price Processes

by

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Abstract

The purpose of this paper is to study a large trader's impact on the price process. Maintaining the usual assumptions of complete and frictionless markets, we model a pure exchange economy where two types of *symmetrically* informed agents exist: the *large trader*, interacting with the other type of agent, so-called *reference traders*. The large trader chooses his policy such that his expected utility from terminal wealth is maximized.

It turns out that drift and volatility of the risky asset's return change *stochastically* according to the large trader's trading strategy while these coefficients remain constant in the same economy without a large trader. We show *how* the stock's drift and volatility depend on the large trader's trading strategy which in turn depends largely on his risk aversion. Furthermore, we present upper und lower bounds for the stochastic drift and volatility coefficients. Surprisingly the large trader's wealth process is *independent* of his impact on the price process.

(JEL-Classification: C60, D40, D51, D90, G11, G12)

Contents

1 Introduction

A non-price-taking investor influences market prices since his order volume is large compared to the order volume of price-taking market participants. Therefore a non-price-taking investor is often called *large trader* or large investor. The purpose of this paper is to study a large trader's impact on the price process. We are interested in the large investor's optimal terminal wealth, his optimal investment policy as well as in the effects of this policy on the price process.

Up to now, almost all models describing stock markets assume price-taking investors. However, an observation of today's security markets reveal the everincreasing importance of financial institutions in the market-place. Large investors, i.e. agents wealthy enough to make it worthwile taking account of the effect of their tradings on the market, are particularly prevalent in smaller security markets. A large investor may have a significant effect on prices and hence may prefer to choose a strategy taking the price impact of his own behavior into account. Therefore it is reasonable to develop a model describing stock markets containing a large trader and to analyze the consequences of his actions on prices.

Our approach to this problem is to model a pure exchange economy, maintaining the usual assumptions of complete and frictionless markets and *symmetric* information. The economy considered is based on two types of agents: the *large trader*, interacting with the other type of agents, so-called *reference traders*, such as uninformed speculators. The effect of the large investor's presence in the market is then studied by comparing equilibrium prices in the reference economy, i.e. the economy without the large trader, to the prices in an economy where the large trader is active on the market. We assume, the large trader chooses his policy such that his expected utility from terminal wealth is maximized. We switch to a dual optimization problem and adopt martingale methods (Pliska (1986), Karatzas, Lehoczky and Shreve (1987), Cox und Huang (1989, 1991)) making the analysis highly tractable.

Rational expectation models, like the one of Brennan and Schwartz (1989), assume that reference traders are only concerned about the long-term prospects of the risky asset. Since the risky asset's terminal value is entirely determined by an exogenously given random variable which is interpreted as the fundamental value of the asset, agents' expectations are solely driven by the gradually revealed information about the value of this state variable. In particular agents do not alter their expectations in reaction to changes in the current price. Empirical evidence contrasts, however, with this demanding assumption on agents' rationality in this Radner-type equilibrium approach. When making trading decisions for very short periods, like in intraday dealing, *investors seem to rely more on the information conveyed by current price movements than on the long-term fundamental prospects of the assets.* In fact there seems to be a *positive feedback effect* of current price changes on expectations, theoretically justified for instance by De Long, Shleifer, Summers, and Waldmann (1990). This *extrapolative* way of expectation formation is a central ingredient of the Frey/Stremme (1997) model which serves as basis of the reference model of this paper.

Recent studies, modeling the large trader in a continuous-time setting are those of Cuoco and Cvitanic (1996), Cvitanic (1997a,b) and El Karoui, Peng and Quenez (1997). These models exogenously specify a price dependence on the large investor's policy and do not require market clearing. Cvitanic solves the large trader's optimal *investment* problem while Cuoco and Cvitanic analyze the large trader's optimal *consumption* problem. But both papers only allow the large traders policy to affect the instantaneous drift but not the instantaneous volatility of the security price. Cvitanic and Ma (1996) assume that the drift and volatility functions can both be nonlinear in the price process and also depend on the wealth process and the portfolio process of the large investor and consider the problem of hedging contingent claims but *without* imposing any equilibrium conditions. They use the theory of forward-backward stochastic differential equations and have to impose quite hard conditions on the drift and diffusion terms.

Lindenberg (1979) was perhaps the first in analyzing the large investor's optimal consumption problem in a discrete two period equilibrium model. Basak (1997) considers in a discrete time model the optimal consumption choice of a large investor and the implications on asset prices in a price-leadership model that is consistent with rational expectations. Working also in discrete time, Jarrow (1992, 1994) exogenously specifies a dependence of asset prices on the non-price-taker's trading strategy, and focuses on market manipulation strategies which generate arbitrage opportunities for the non-price-taker.

The present paper is organized as follows: In section 2 we explain the economy. Section 3 contains the large investor's optimal investment policy in a general setting which is then, in section 4, applied to the model of section 2 to analyze the impact of the large trader's optimal investment policy on the price process. Section 5 concludes the paper. Appendix A contains the theory of dynamic replication of attainable contingent claims in an economy where asset prices are influenced by the large trader's wealth and policy, which is used in section 3. Proofs can be found in Appendix B.

2 The Economy

2.1 Discrete Reference Economy

Consider a pure exchange economy with an infinite number of trading dates $t = 0, 1, 2, \ldots$, where a risky asset (stock) and a riskless asset (bond) are traded. The price of the risky asset, S, is expressed in units of the price of the safe asset. That is, the price of the safe asset serves as a numeraire for the price of the risky asset. Moreover, idealizing the fact that the bond market is far more liquid than that for the risky asset, we suppose the market for the riskless asset to be perfectly elastic.

For every t there is a generation of agents born, living for two periods. The young agents born at time t receive an exogenously given stochastic wealth Y_t and determine their demand for stock and money market account by maximizing the expected utility of next periods wealth. At time t, every agent $a \in A$ forms his demand for the risky asset as a function of wealth Y_t^a and the proposed Walrasian price s:

$$D_t(Y_t^a, s) \quad = \quad \arg\max_{d \geq 0} E\left[u\left(Y_t^a + d \cdot (\tilde{S}_{t+1} - s)\right)\right], \tag{1}$$

where $\tilde{S}_{t+1}$ denotes the expected next periods price. Their holdings of the money market account are determined by the budget identity. The equilibrium asset price S_t at time t is then defined by the market clearing equation:

$$\sum_{a \in A} D_t(Y_t^a, S_t) = 1, \tag{2}$$

where we have normalised the total supply of stock to unity. Old agents living already one period at time t, simply consume and disappear from the market. That is, we are working in an *overlapping generations model without bequest and without first period consumption.* It is a stylized model of a market where agent's investment decisions are made sequentially over time and where each decision is determined mainly by short-term considerations. Market participants might be managers from investment funds who are managing a fluctuating, exogenously given amount of assets. Typically these investors are (at least partly) compensated according to the performance of their portfolio. This performance is usually evaluated at certain dates such that the investment decisions of these agents are often aimed at the next evaluation date.

Tastes of Agents

Now we introduce our assumptions on utility functions and expectations of our reference traders. Since our focus is not on aggregation problems, we simply assume the existence of a *representative reference trader* (in the following called reference trader). His utility function u exhibits constant relative risk aversion γ, i.e.

$$u(s) = \begin{cases} \frac{1}{1-\gamma} s^{1-\gamma} & \text{with } \gamma > 0, \gamma \neq 1 \\ \ln(s) & \text{for } \gamma = 1 \end{cases} \tag{3}$$

and

$$u'(s) = s^{-\gamma}. \tag{4}$$

Beliefs of Agents

We assume that the reference trader takes the proposed price s as a signal and determines the expected next periods price $\tilde{S}_{t+1}$ as

$$\tilde{S}_{t+1} = s \cdot \varepsilon_{t+1} \ , \tag{5}$$

where $\{\varepsilon_t\}_{t \in I\!N}$ is a sequence of independent and identically distributed positive random variables with $E(\varepsilon_t) \geq 1$. Note that there is a *positive feedback* from the proposed price into agents expectations: after a rise of s, the reference trader anticipates a rise of the future price and in the case of a price decline he expects the future price to go down.

Equilibrium

With (1) the reference trader's demand for stock is now given by

$$\begin{aligned} D_t(y, s) &= \arg\max_{d \geq 0} E\left[u\left(y + d \cdot (\tilde{S}_{t+1} - s)\right)\right] \\ &= \arg\max_{d \geq 0} E\left[u\left(y + d \cdot s \cdot (\varepsilon_{t+1} - 1)\right)\right]. \end{aligned} \tag{6}$$

Since $\{\varepsilon_t\}_{t\in I\!N}$ are identically distributed, the demand function is independent of time. The first order condition for this portfolio problem,

$$E[(y + d \cdot s \cdot (\varepsilon_{t+1} - 1))^{-\gamma} \cdot s \cdot (\varepsilon_{t+1} - 1)] = 0,$$

implies that the demand function D is homogenous of degree one w.r.t. the wealth y :

$$D(\alpha \cdot y, s) = \alpha \cdot D(y, s) \quad \text{for all } \alpha \geq 0.$$

It is also homogenous of degree zero w.r.t. the wealth and stock price (y, s):

$$D(\alpha \cdot y, \alpha \cdot s) = D(y, s) \quad \text{for all } \alpha \geq 0.$$

It follows that D is of the form:

$$D(y, s) = \frac{y}{s} \cdot D(1, 1) \tag{7}$$

where $D^* := D(1, 1)$ is the reference trader's demand of stock if his wealth and current stock price both are one. Since $D^* > 0$ the reference trader's demand function is decreasing in the stock price s and increasing in wealth y.

We further assume that the process of the reference trader's wealth is of the form

$$Y_{t+1} = Y_t \cdot exp\left\{(\alpha_y - \frac{1}{2}\sigma_y^2)\Delta t + \sigma_y \sqrt{\Delta t}\, \xi_{t+1}\right\} \tag{8}$$

where $\{\xi_t\}_{t\in I\!N}$ is an i.i.d. sequence of standard-normally distributed random variables.

Since in equilibrium $\frac{Y_t}{S_t} \cdot D^* = 1$ must hold, the equilibrium stock price process fulfills

$$\begin{aligned} S_{t+1} &= D^* \cdot Y_{t+1} = D^* \cdot Y_t \cdot exp\left\{(\alpha_y - \frac{1}{2}\sigma_y^2)\Delta t + \sigma_y \sqrt{\Delta t}\, \xi_{t+1}\right\} \\ &= S_t \cdot exp\left\{(\alpha_y - \frac{1}{2}\sigma_y^2)\Delta t + \sigma_y \sqrt{\Delta t}\, \xi_{t+1}\right\}. \end{aligned} \tag{9}$$

That is, the equilibrium stock price process is driven by the same process as the wealth process.

Note that reference trader's expectations are rational if and only if

$$\varepsilon_t = exp\left\{(\alpha_y - \frac{1}{2}\sigma_y^2)\Delta t + \sigma_y \sqrt{\Delta t}\, \xi_{t+1}\right\} \quad \text{for all } t \in I\!N.$$

We now take a non-price taking investor into consideration.

Equilibrium with a large investor
If we denote the large investor's wealth with X and the proportion of his wealth invested in the stock with π, then the function of total demand for the risky asset has the following representation:

$$G(t, y, s, \pi, x) = D^* \cdot \frac{y}{s} + \frac{\pi x}{s}$$

Market clearing requires then that

$$G(t, Y_t, S_t, \pi_t, X_t) = D^* \cdot \frac{Y_t}{S_t} + \frac{\pi_t X_t}{S_t} = 1. \tag{10}$$

2.2 Continuous Time Economy

In order to get a clearer picture of the effect the large trader's action has on the equilibrium stock price, we will now reduce the intervalls between subsequent trading dates and pass to the continuous time model. This brings us closer to continuous time optimization models. The details of this transition to the limit can be found in Frey and Stremme (1997, p.360).

In the following we assume that the process of the reference trader's wealth fulfills

$$dY_t = Y_t \alpha_y dt + Y_t \sigma_y dW_t \,, \quad 0 \le t \le T, \tag{11}$$

that is for $t \in [0, T]$ we have $Y_t = Y_0 exp\{(\alpha_y - \frac{1}{2}\sigma_y^2)t + \sigma_y W_t\}$. The equilibrium stock price of the reference model is then given by

$$dS_t = S_t \alpha_y dt + S_t \sigma_y dW_t \,, \quad 0 \le t \le T, \tag{12}$$

which is equivalent to

$$S_t = S_0 exp \left\{ (\alpha_y - \frac{1}{2}\sigma_y^2)t + \sigma_y W_t \right\}, \quad 0 \le t \le T.$$

The large trader's wealth process is the solution of the stochastic differential equation

$$dX_t = \pi_t X_t \frac{1}{S_t} dS_t \tag{13}$$

with $X_0 = x$.

For simplicity we assume that the large trader's strategy is constant. Since (10) is equivalent to $S_t = D^*Y_t + \pi_t X_t$ we get with (11) and (13)

$$(1 - \pi_t^2 X_t \frac{1}{S_t})\, dS_t = D^*Y_t\alpha_y dt + D^*Y_t\alpha_y dW_t, \quad 0 \le t \le T.$$

The division by $1 - \pi_t^2 X_t \frac{1}{S_t}$ yields an representation of the form

$$dS_t = S_t\alpha_t dt + S_t\sigma_t dW_t$$

with

$$\alpha_t = \frac{D^*Y_t}{S_t - \pi_t^2 X_t}\alpha_y$$

and

$$\sigma_t = \frac{D^*Y_t}{S_t - \pi_t^2 X_t}\sigma_y.$$

So by using again the market clearing equation to replace the stock price in the denominator we get that the equilibrium stock price process satisfies the stochastic differential equation

$$dS_t = S_t\alpha(t,\omega,\pi_t,X_t)dt + S_t\sigma(t,\omega,\pi_t,X_t)dW_t, \quad 0 \le t \le T, \tag{14}$$

with $S_0 = D^*Y_0 + \pi_0 X_0$ and the drift and diffusion parameters being defined as

$$\alpha(t,\omega,\pi_t,X_t) = \begin{cases} \alpha_y, & X_t \le 0 \\ \frac{D^*Y_t}{D^*Y_t + \pi_t(1-\pi_t)X_t}\alpha_y, & X_t > 0,\ D^*Y_t + \pi_t(1-\pi_t)X_t \ne 0,\ \pi_t X_t + D^*Y_t > 0 \\ 0, & \text{else.} \end{cases} \tag{15}$$

$$\sigma(t,\omega,\pi_t,X_t) = \begin{cases} \sigma_y, & X_t \le 0 \\ \frac{D^*Y_t}{D^*Y_t + \pi_t(1-\pi_t)X_t}\sigma_y, & X_t > 0,\ D^*Y_t + \pi_t(1-\pi_t)X_t \ne 0,\ \pi_t X_t + D^*Y_t > 0 \\ 0, & \text{else.} \end{cases} \tag{16}$$

So we have an explicit expression for the influence of the large trader's portfolio process and wealth process on the drift and diffusion coefficient of the price process. If the portfolio process is constant, then the solution of the differential equation (14) with the drift and diffusion coefficients (15) and (16), respectively, is the stock price of an equilibrium model.

3 Investment Policy of a Large Trader

We now determine the large investor's optimal trading strategy, i.e. that policy π which maximizes his expected utility from terminal wealth X_T^π :

$$\text{(PP)} \qquad \max_{\pi \in A(x)} EU(X_T^\pi) \tag{17}$$

The optimal policy is calculated for diffusion price processes with arbitrary drift and diffusion coefficients and then specialized to the situation where these coefficients are given by equations (15) and (16).

3.1 Arbitrary diffusion price processes

As before, we consider a continuous time economy with finite time horizon T where two assets, a riskless asset (bond, serving as the numeraire) and a risky asset (stock), are traded. The stock price process is given by the stochastic differential equation

$$dS_t = S_t \alpha_t dt + S_t \sigma_t dW_t, \; 0 \leq t \leq T, \quad S_0 \text{ given}, \tag{18}$$

where $\{W_t\}_{t \in [0,T]}$ is a standard Brownian motion on a complete probability space (Ω, F, P) with the augmentation under P of the filtration $\mathcal{F} = \{F_t\}_{t \in [0,T]}$ generated by the Brownian motion W. All stochastic processes encountered throughout this paper are progressively measurable with respect to $\mathcal{F}$. The drift and diffusion parameters α_t and σ_t, respectively, of the stock price are of the form

$$\alpha_t = \alpha(t, \omega, \pi_t, X_t), \tag{19}$$

$$\sigma_t = \sigma(t, \omega, \pi_t, X_t), \tag{20}$$

where as before X_t and π_t denote the large investor's wealth and the proportion of wealth invested in the stock at time t, respectively, and $\alpha : [0, T] \times \Omega \times I\!R^2 \rightarrow I\!R$ and $\sigma : [0, T] \times \Omega \times I\!R \rightarrow I\!R$ are arbitrary functions. Furthermore, we set $\overline{\alpha}_t := \alpha(t, \omega, 0, 0)$ and $\overline{\sigma}_t := \sigma(t, \omega, 0, 0)$. We assume that the large trader is endowed with initial wealth $x > 0$ and no holdings in the stock.

Definition 1

A *trading strategy* or *portfolio process* is any progressivly measurable, $I\!R-$ valued process π with

$$\int_0^T |\,\pi_t X_t \alpha_t\,|\; dt + \int_0^T (\pi_t X_t \sigma_t)^2 \; dt < \infty. \tag{21}$$

Using a trading strategy π, the large investor's wealth process $X = X^{x,\pi}$ then satisfies the stochastic differential equation

$$dX_t = \pi_t X_t \alpha_t dt + \pi_t X_t \sigma_t dW_t, \quad 0 \le t \le T,\ X_0 = x. \tag{22}$$

Definition 2

A trading strategy is called *admissible* for initial wealth $x > 0$ if $X_t^{x,\pi} \ge 0$ holds for all $t \in [0,T]$ almost surely. For initial wealth $x > 0$ we denote the set of admissible trading strategies by $A(x)$.

Assumption 1

The large trader's utility function U is a strictly increasing, strictly concave, continuously differentiable function $U : (0,\infty) \to I\!R$ with

$$\lim_{x\to 0} U'(x) = \infty, \quad \lim_{x\to\infty} U'(x) = 0.$$

If the large trader's utility function U is of the HARA class with coefficient $\gamma > 0$, then U satisfies this assumption. The conditions of the assumption guarantee that the marginal utility function U' has a continuous, strictly decreasing inverse $I : (0,\infty) \to (0,\infty)$. We introduce the function

$$\tilde{U}(y) := \max_{x>0}\{U(x) - xy\} = U(I(y)) - yI(y). \tag{23}$$

As a direct consequence we get the useful inequality

$$U(I(y)) \ge U(x) + y(I(y) - x), \quad x, y > 0. \tag{24}$$

We generalize the definition of the risk premium process and define the process

$$\kappa_\lambda(t) \;\; := \;\; \frac{\overline{\alpha}_t}{\overline{\sigma}_t} + \lambda^1(t), \tag{25}$$

where $\lambda = (\lambda^0, \lambda^1) \in I\!R^2$ is a bounded, twodimensional process. This definition takes into account that the risk premium depends on the large trader's portfolio process and wealth process and therefore can be different from the risk premium $\frac{\overline{\alpha}_t}{\overline{\sigma}_t}$ of the reference model without a large trader.

Furthermore, we define

$$Z_\lambda(t) \;:=\; \exp\left\{-\int_0^t \kappa_\lambda(s)dW_s - \frac{1}{2}\int_0^t \kappa_\lambda^2(s)ds\right\}, \tag{26}$$

the discount processes

$$\beta_\lambda(u,t) \;:=\; \exp\left\{\int_u^t \lambda^0(s)ds\right\} \tag{27}$$

$$\beta_\lambda(t) \;:=\; \beta_\lambda(0,t) \tag{28}$$

and the stochastic deflator process

$$H_\lambda(t) \;:=\; Z_\lambda(t)\beta_\lambda(t). \tag{29}$$

for bounded processes $\lambda = (\lambda^0, \lambda^1)$.

By Ito's lemma we get

$$\begin{aligned} H_\lambda(T)X_T &= x + \int_0^T H_\lambda(t)X_t\big(\pi_t\alpha_t - \lambda^0(t) - \kappa_\lambda(t)\pi_t\sigma_t\big)dt \\ &\quad + \int_0^T H_\lambda(t)X_t\big(\pi_t\sigma_t - \kappa_\lambda(t)\big)dW_t. \end{aligned}$$

If we define the nonnegative function $\tilde{g}(t,\omega,\pi,x) : [0,T]\times\Omega\times I\!R^2 \to I\!R$ as

$$\begin{aligned} \tilde{g}(t,\omega,\lambda) &:= \sup_{(\pi,x)\in I\!R^2}\Big[\pi x\alpha(t,\omega,\pi,x) - \lambda^0 x - \kappa_\lambda \pi x\sigma(t,\omega,\pi,x)\Big] \\ &= \sup_{(\pi,x)\in I\!R^2}\left[\pi x\alpha_t - x\lambda^0 - \pi x\sigma_t\left(\frac{\overline{\alpha}_t}{\overline{\sigma}_t} + \lambda^1\right)\right], \end{aligned} \tag{30}$$

then $\tilde{g}$ gives the supremum of the drift of HX and will be useful for further calculations.

Assumption 2

The function $\tilde{g}(t,\cdot)$ is convex.

Under this assumption, the *effective domain* $\mathcal{N}_t := \{\lambda \in \mathcal{R}^2 : \tilde{g}(t,\lambda) < \infty\}$ of $\tilde{g}(t,\cdot)$ is bounded and convex.[1] The family of twodimensional processes λ, satisfying $\lambda_t \in \mathcal{N}_t$ for all $t \geq 0$, is denoted by $\mathcal{N}$.

We impose the following assumption:

Assumption 3

The function $\tilde{g}(t,\cdot)$ is bounded on its effective domain, uniformly in t. The sets $\mathcal{N}_t$ are bounded uniformly. Further, the set $\mathcal{N}$ is not empty.

Since $\mathcal{N}$ is bounded, the process Z_λ is a martingale for any $\lambda \in \mathcal{N}$. Therefore by

$$P^\lambda(A) := E[Z_\lambda(T)1_A] = E^\lambda[A], \quad A \in \mathcal{F}_T$$

a probability measure is defined and due to the Girsanov theorem the process

$$W_\lambda(t) := W_t - \int_0^t \kappa_\lambda(s)ds$$

is a P^λ -Brownian motion.

Now we are able to formulate the following proposition about the relation between the primal optimization problem (17) and the dual optimization problem

$$\text{(DP)} \qquad \inf_{\lambda \in \mathcal{N}} E\left[\tilde{U}(yH_\lambda(T)) + y\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right] \qquad (31)$$

where y is a positive real constant.

Proposition 1

(a) Let y be a positive, real constant and $\pi \in A(x)$ be an admissible strategy for a large trader with initial wealth $x > 0$. Then the following inequality holds for all processes $\lambda \in \mathcal{N}$:

$$EU(X_T^\pi) \leq yx + E\left[\tilde{U}(yH_\lambda(T)) + y\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right]. \qquad (32)$$

(b) If for the pair $(\hat{\pi},\widehat{\lambda}) \in A(x) \times \mathcal{N}$ inequality (32) holds as an equality, then the strategy $\hat{\pi}$ is a solution of the primal optimization problem (17) and $\widehat{\lambda}$ is a solution of the dual optimization problem (31).

[1] See Rockafellar (1974).

(c) Relation (32) holds as an equality if and only if the following three conditions are satisfied:

$$\begin{aligned} \hat{X}_T &= I(yH_{\widehat{\lambda}}(T)) & (33)\\ \tilde{g}(t,\widehat{\lambda}_t) &= \hat{\pi}_t\hat{X}_t\alpha_t + \hat{X}_t\widehat{\lambda}^0(t) + \hat{\pi}_t\hat{X}_t\sigma_t\kappa_{\widehat{\lambda}}(t) & (34)\\ E[H_{\widehat{\lambda}}(T)\hat{X}_T] &= x + E\left[\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt\right] & (35) \end{aligned}$$

PROOF: see Appendix B.

This proposition suggests the following procedure for solving the primal optimization problem:

STEP 1: For any $y > 0$ determine a solution $\widehat{\lambda}_y$ of the dual problem.

STEP 2: (a) For any $y > 0$ and the contingent claim $B = I(yH_{\widehat{\lambda}_y}(T))$ determine the arbitrage-free price $h(0) = h_y(0)$ and a replication strategy $\pi \in A(h_y(0))$.

(b) For the initial wealth x determine $y = y_x$ such that $x = h_y(0)$.

So we divide our problem mainly into two parts. First, we determine the optimal terminal wealth and then we determine a strategy which replicates this terminal wealth.

All results concerning the theory of replicating a contingent claim in an economy where the stock price is effected by a non-price-taking-agent's wealth and policy can be found in Appendix A.

Proposition 2

Let $\widehat{\lambda} = \widehat{\lambda}_y$ be a solution of the dual optimization problem (31) and let $y = y(x)$ be such that

$$x = h_y(0) = E^{\widehat{\lambda}}\left[\beta_{\widehat{\lambda}}(T)B - \int_0^T \beta_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt\right].$$

(a) The large investor's optimal terminal wealth is given by $\hat{X}_T = I(yH_{\widehat{\lambda}}(T))$. For the optimal wealth process the following holds:

$$X_t = E^{\widehat{\lambda}}\left[X_T\beta_{\widehat{\lambda}}(T) - \int_t^T \beta_{\widehat{\lambda}}(s)\tilde{g}(s,\widehat{\lambda}_s)ds\middle|\mathcal{F}_t\right]$$

(b) The portfolio process $\hat{\pi}$, satisfying

$$\hat{\pi}_t \hat{X}_t \, \sigma(t, \hat{\pi}_t, \hat{X}_t) \quad = \quad \psi_{\widehat{\lambda}}(t) \beta_{\widehat{\lambda}}^{-1}(t),$$

is a solution of the primal optimization problem (17). The process $\psi_{\widehat{\lambda}}$ denotes the integrand in the representation of the $P^{\widehat{\lambda}}$-martingale $M_{\widehat{\lambda}}(t) := V_t \beta_{\widehat{\lambda}}(t) - \int_0^t \beta_{\widehat{\lambda}}(s) \tilde{g}(s, \widehat{\lambda}(s)) ds$ as a stochastic integral $dM_{\widehat{\lambda}} = \psi_{\widehat{\lambda}} dW_{\widehat{\lambda}}$ with $V_t = \operatorname{ess\,sup}_{\lambda \in \mathcal{N}} E^{\lambda} \left[X_T \beta_\lambda(t, T) - \int_t^T \beta_\lambda(t, s) \tilde{g}(s, \lambda_s) \, ds \middle| F_t \right]$.

3.2 A specific diffusion price processes

We now return to the model of section 2 and apply the results of the previous subsection. First, we compute the function $\tilde{g}$. It is easy to show that

$$\begin{aligned} \tilde{g}(t, \lambda) &= \sup_{(\pi, x) \in \mathbb{R}^2} [\pi x \alpha(t, \pi, x) - x\lambda^0 - \pi x \sigma(t, \pi, x) \left(\frac{\overline{\alpha}_y}{\overline{\sigma}_y} + \lambda^1 \right)] \\ &= \begin{cases} 0 & , \lambda^0 = 0, \lambda^1 = 0 \\ \infty & , \text{otherwise} \end{cases} \end{aligned}$$

for all $t \in [0, T]$. So the effective domain of $\tilde{g}(t, \cdot)$ is a singleton for all $t \in [0, T]$

$$\begin{aligned} N_t &= \{(\lambda^0, \lambda^1) \in \mathbb{R}^2 : \tilde{g}_t < \infty\} \\ &= \{(0, 0)\}. \end{aligned}$$

Hence, the set $\mathcal{N}$ only consists of the two-dimensional process $\widehat{\lambda}$ with $\widehat{\lambda}_t = (0, 0)$ for all $t \in [0, T]$. For any $y > 0$ the dual optimization problem

$$\inf_{\lambda \in \mathcal{N}} E \left[\tilde{U}(y H_\lambda(T)) + y \int_0^T H_\lambda(t) \tilde{g}(t, \lambda_t) dt \right]$$

has the unique solution $\widehat{\lambda}$. Then the large investor's optimal terminal wealth is again given by equation (33) and we simply have to compute that y for which the arbitrage-free price of the claim $I(y H_{\widehat{\lambda}}(T))$ equals the initial wealth x. That is y has to satisfy the condition

$$x = h_y(0) = E^{\widehat{\lambda}} \left[\beta_{\widehat{\lambda}}(T) I(y H_{\widehat{\lambda}}(T)) - \int_0^T \beta_{\widehat{\lambda}}(t) \tilde{g}(t, \widehat{\lambda}_t) dt \right]$$

which reduces to

$$x = h_y(0) = E^{\widehat{\lambda}} \left[\beta_{\widehat{\lambda}}(T) I(y H_{\widehat{\lambda}}(T)) \right]$$

since $\tilde{g}(t, \widehat{\lambda}) \equiv 0$. If the large trader's utility function U is of the HARA class with coefficient $\gamma > 0$, then U satisfies the conditions imposed on the utility function in section 3 and for any initial wealth $x > 0$ the equation above has a unique solution y. Further, for these utility functions, the inverse I of U' is given by

$$I(y) = y^{-\frac{1}{\gamma}}. \tag{36}$$

The following proposition describes the large trader's optimal trading strategy.

Proposition 3 (Optimal investment policies)

Let the large trader's utility function U be from the HARA class. Then the following two statements hold:

(a) There exists an optimal trading strategy given by

$$\hat{\pi}_t^L = \frac{1}{2}\left(1 - \gamma \frac{\sigma_y^2}{\alpha_y} \frac{Y_t}{\hat{X}_t} D^*\right) + \frac{1}{2}\sqrt{\left(1 - \gamma \frac{\sigma_y^2}{\alpha_y} \frac{Y_t}{\hat{X}_t} D^*\right)^2 + 4D^* \frac{Y_t}{\hat{X}_t}}$$

for all $t \in [0, T]$. For $\gamma \geq \alpha_y / \sigma_y^2$ we have $0 \leq \hat{\pi}_t^L \leq 1$ and for $\gamma < \alpha_y / \sigma_y^2$ we have $\hat{\pi}_t^L > 1$. Hence, he holds a long position in the risky asset.

(b) There exists a second optimal investment strategy given by

$$\hat{\pi}_t^S = \frac{1}{2}\left(1 - \gamma \frac{\sigma_y^2}{\alpha_y} \frac{Y_t}{\hat{X}_t} D^*\right) - \frac{1}{2}\sqrt{\left(1 - \gamma \frac{\sigma_y^2}{\alpha_y} \frac{Y_t}{\hat{X}_t} D^*\right)^2 + 4D^* \frac{Y_t}{\hat{X}_t}}$$

if $\gamma < \alpha_y / \sigma_y^2$. Since $\hat{\pi}_t^S < 0$, he holds a short position in the risky asset.

PROOF: see Appendix B.

Figures 1 illustrates the large trader's optimal strategy for different risk aversion parameters γ. Panel A contains a sample path of the large trader's investment strategy when his risk aversion is relatively strong with $\gamma = 2$. It is easy to see that the fraction of wealth invested in the risky asset at time t, $\hat{\pi}_t^1$, is always above 100% and always below 125%. The trading strategy's upper bound of 125% corresponds to the optimal trading strategy in the reference model. Panel B depicts a possible sample path of the optimal investment strategy when the large trader exhibits risk aversion with $\gamma = 1$ (log-utility). The strategy $\hat{\pi}^1$

oscillates quite heavily between the lower bound of 100% and the upper bound of 250%.

Now we compare the large trader's optimal terminal wealth with the terminal wealth he could reach in the reference economy where his trades have no effect on the stock price. In the latter case the stock price process is a geometric Brownian motion satisfying

$$dS_t = S_t \alpha_y dt + S_t \sigma_y dW_t, \qquad 0 \leq t \leq T.$$

In this case we find

$$\begin{aligned} \tilde{g}(t, \lambda) &= \sup_{\pi, x} [\pi x \alpha_y - x \lambda^0 - \pi x \sigma_y \left(\frac{\overline{\alpha}_y}{\overline{\sigma}_y} + \lambda^1 \right)] \\ &= \begin{cases} 0 & , \lambda^0 = 0, \lambda^1 = 0 \\ \infty & , \text{otherwise.} \end{cases} \end{aligned}$$

that is the function $\tilde{g}$ is the same as in the case studied before. Therefore the sets $\mathcal{N}_t$ and $\mathcal{N}$ also are the same and the dual optimization problem has also the same solution $\widehat{\lambda}$. The large trader's optimal policy now is given as

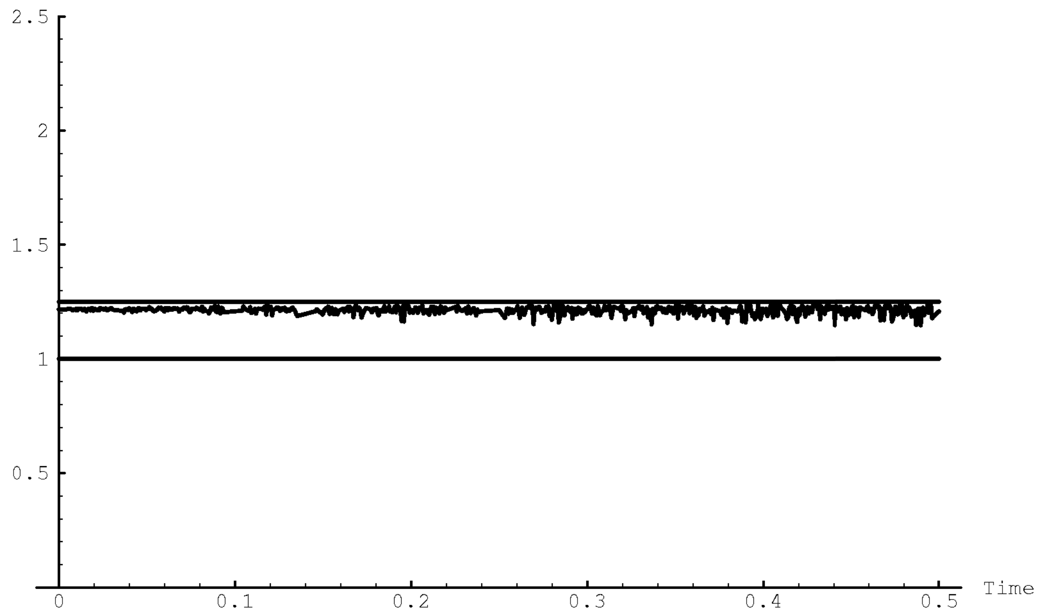
$$\tilde{\pi}_t = \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2}$$

for all $t \in [0, T]$. From equation (50) of the proof of Proposition 3 it follows that $\tilde{\pi}_t > \hat{\pi}_t^L > 1$ if $\gamma < \alpha/\sigma^2$ and that $\tilde{\pi}_t \leq \hat{\pi}_t^L \leq 1$ if $\gamma \geq \alpha/\sigma^2$.

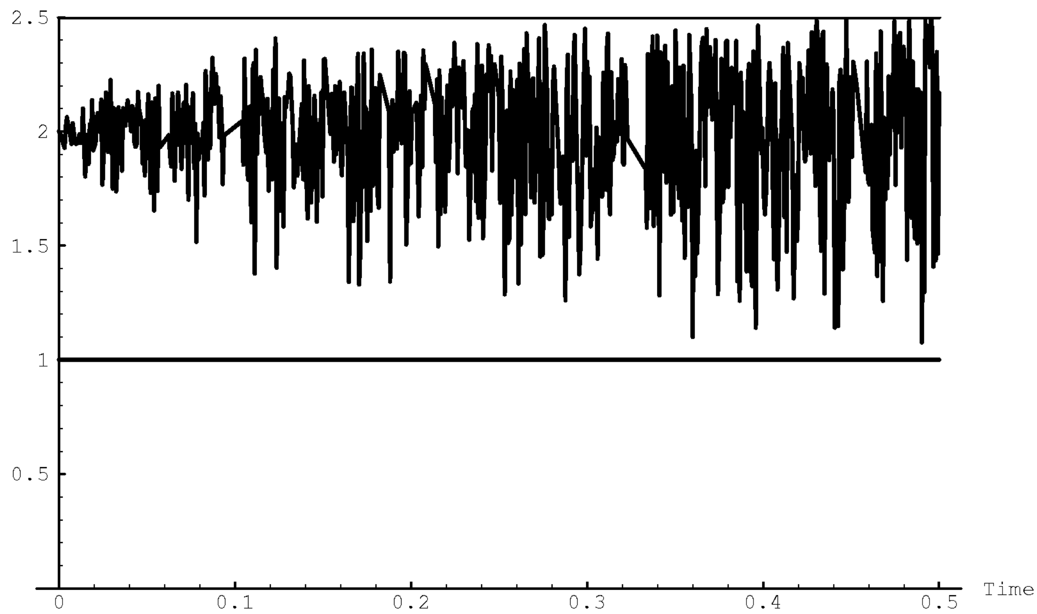
Figure 1: **Optimal investment strategy $\hat{\pi}^L$ and its boundaries**

Parameters: $X_0 = 10$ $Y_0 = 100$ $D^* = 1$ $\alpha_y = 0.1$ $\sigma_y = 0.2$

Panel A: $\gamma = 2$ (the large trader's relative risk aversion)



Panel B: $\gamma = 1$ (the large trader's relative risk aversion)



The next result follows immediately from the fact that the dual problem has the same solution in both economies.

Proposition 4

The large trader's wealth process is unique and independent of his impact on the price process.

PROOF: Since $\hat{\pi}_t \sigma(t, \hat{\pi}_t, \hat{X}_t) = \frac{1}{\gamma}\frac{\alpha_y}{\sigma_y} = \tilde{\pi}\sigma_y$ and $\alpha_t = \frac{\alpha_y}{\sigma_y}\sigma_t$ (the last relation will be shown in the next section) the stochastic differential equations

$$d\hat{X}_t = \hat{X}_t \hat{\pi}_t \alpha(t, \hat{\pi}_t, \hat{X}_t)dt + \hat{X}_t \hat{\pi}_t \sigma(t, \hat{\pi}_t, \hat{X}_t)dW_t$$

and

$$dX_t = X_t \tilde{\pi}_t \alpha_y dt + X_t \tilde{\pi}_t \sigma_y dW_t$$

have the same solution. □

So we have found that the large trader has neither an advantage nor a disadvantage from the fact that his trades influence the stock price. This result holds for any utility function that satisfies the conditions imposed on the utility function in section 3.

4 A Large Trader's Impact on Price Processes

In this section we analyze the impact of the large trader's optimal investment policy, which we have deduced in the previous section, on price processes. First, we compare the risk premium of the stock price in the model where the large trader is active, with the risk premium of the stock price in the reference economy.

Proposition 5 (Risk premium)

The risk premium equals that in the reference economy, that is we have:

$$\frac{\alpha_t}{\sigma_t} = \frac{\alpha_y}{\sigma_y}, \quad 0 \leq t \leq T.$$

Proof: The statement is true since in equilibrium the drift and diffusion parameters α and σ satisfy

$$\begin{aligned}
\alpha(t, \omega, \hat{\pi}_t, \hat{X}_t) &= \frac{D^* Y_t}{D^* Y_t + \hat{\pi}_t(1 - \hat{\pi}_t)\hat{X}_t} \alpha_y, \\
\sigma(t, \omega, \hat{\pi}_t, \hat{X}_t) &= \frac{D^* Y_t}{D^* Y_t + \hat{\pi}_t(1 - \hat{\pi}_t)\hat{X}_t} \sigma_y.
\end{aligned}$$

□

Now we consider the drift and diffusion terms of the resulting stock price in detail.

Proposition 6 (Drift and diffusion coefficients)

The drift and diffusion coefficients α and σ, respectively, are of the form

$$\begin{aligned}
\alpha(t, \omega, \hat{\pi}_t, \hat{X}_t) &= \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \frac{1}{\hat{\pi}_t} \alpha_y \\
\sigma(t, \omega, \hat{\pi}_t, \hat{X}_t) &= \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \frac{1}{\hat{\pi}_t} \sigma_y
\end{aligned}$$

for all $t \in [0, T]$.

Proposition 7 (Bounds for drift and diffusion coefficients)

(a) If the proportion of wealth that the large trader invests in the risky asset is is greater than one ($\hat{\pi}_t^L > 1$), upper and lower bounds for the drift and diffusion coefficients are given as follows:

$$\alpha_y \; < \; \alpha(t, \hat{\pi}_t, \hat{X}_t) \; < \; \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \, \alpha_y, \quad 0 \le t \le T,$$

$$\sigma_y \; < \; \sigma(t, \hat{\pi}_t, \hat{X}_t) \; < \; \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \, \sigma_y, \quad 0 \le t \le T.$$

(b) If the proportion of wealth that the large trader invests in the risky asset is between zero and one ($0 \le \hat{\pi}_t^L \le 1$), upper and lower bounds for the drift and diffusion coefficients are given as follows:

$$\alpha_y \; \ge \; \alpha(t, \hat{\pi}_t, \hat{X}_t) \; \ge \; \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \, \alpha_y, \quad 0 \le t \le T,$$

$$\sigma_y \; \ge \; \sigma(t, \hat{\pi}_t, \hat{X}_t) \; \ge \; \frac{1}{\gamma} \frac{\alpha_y}{\sigma_y^2} \, \sigma_y, \quad 0 \le t \le T.$$

(c) If the large trader has a short position in the risky asset ($\hat{\pi}_t^S < 0$), upper and lower bounds for the drift and diffusion coefficients are given as follows:

$$-\frac{1}{\gamma} \frac{\hat{X}_t}{D^* Y_t} \frac{\alpha_y}{\sigma_y^2} \, \alpha_y < \alpha(t, \hat{\pi}_t, \hat{X}_t) < 0$$

$$-\frac{1}{\gamma} \frac{\hat{X}_t}{D^* Y_t} \frac{\alpha_y}{\sigma_y^2} \, \sigma_y < \sigma(t, \hat{\pi}_t, \hat{X}_t) < 0$$

Especially, the expected stock return is negative in the case that the large trader is short in the risky asset.

Proof of propositions 6 and 7: If we use equation (50) then we get

$$\frac{D^* Y_t + \hat{\pi}_t (1 - \hat{\pi}_t) \hat{X}_t}{D^* Y_t} \;=\; 1 + \hat{\pi}_t (1 - \hat{\pi}_t) \frac{\hat{X}_t}{D^* Y_t} \;=\; \gamma \, \frac{\sigma_y^2}{\alpha_y} \hat{\pi}_t$$

and the bounds follow. □

If the large trader's initial wealth tends to zero in comparision to the reference trader's wealth, then the volatility of the model with the large trader tends to the volatility of the reference model. If the large trader's initial wealth tends to infinity in comparision to the refernce trader's wealth, then the large trader holds all risky assets and the volatility is zero.

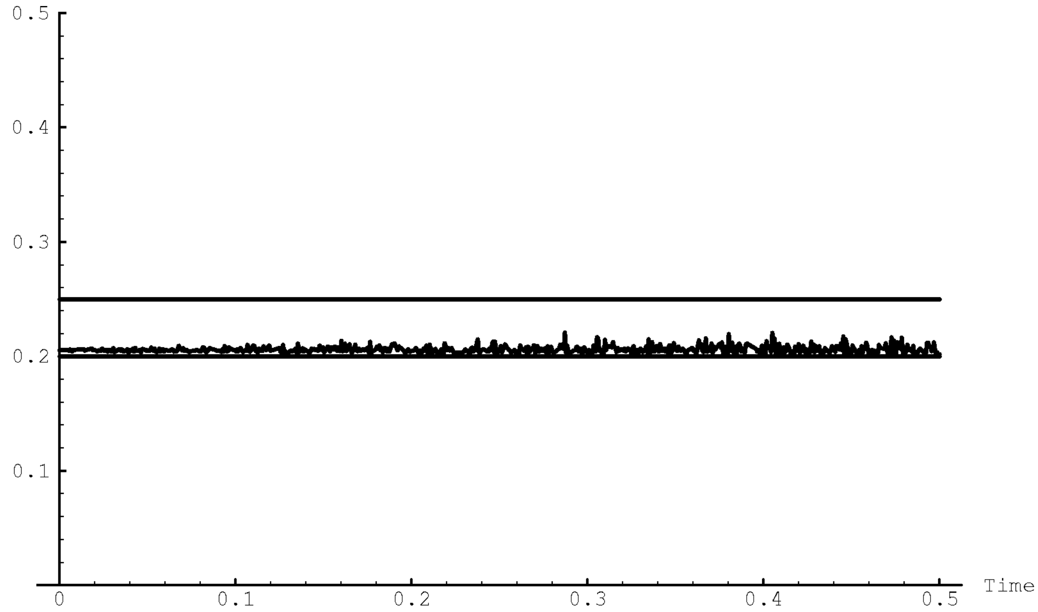
Figures 2 shows two possible sample pathes of stock volatility. The sample path in Panel A corresponds to the situation where the large trader is again relatively risk averse with $\gamma = 2$ and does not deviate to much from its lower bound, namely the volatility of $\sigma_y = 0.2$ in the reference economy. In contrast, the sample path in Panel B oscillates again quite wildly corresponding to the oscillations of the large trader's investment strategy in Panel B of figure 1.

In the previous section, we have seen, that the large trader's wealth process is independent of his impact on stock prices. The stock price processes are not the same. This may be confusing at first glance. The reason for this observation is, that the wealth processes are identical but not his portfolio processes. In the economy where he influences the stock price, his optimal strategy is always smaller than that in the other economy. So on the one hand, the fraction of wealth invested in the risky asset is smaller, but on the other hand the equilibrium stock price and the expected return is higher than in the economy where he does not influence the stock price. These two effects compensate each other so that the wealth processes are the same.

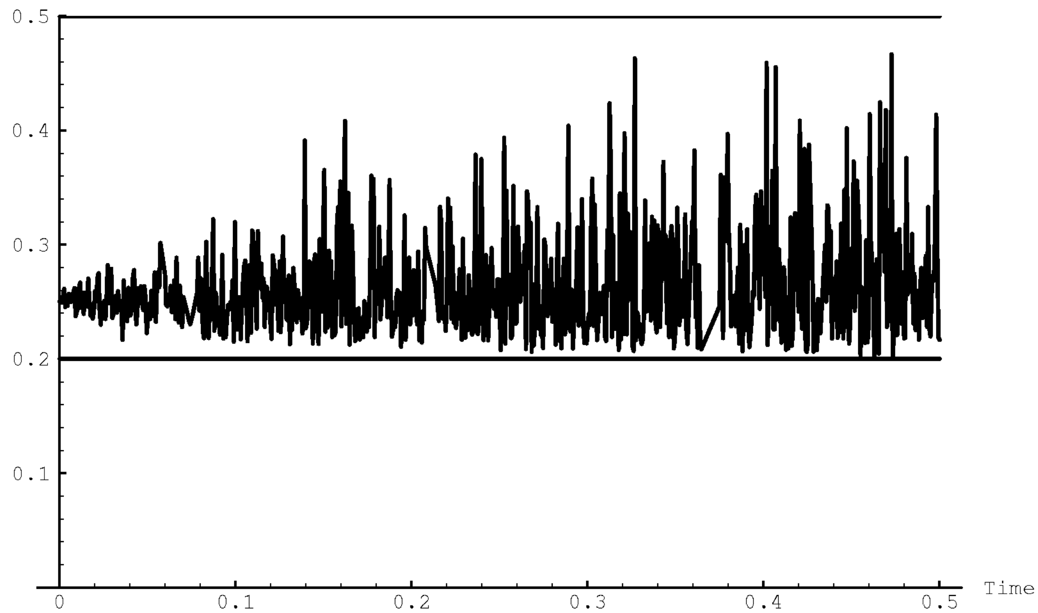
Figure 2: **Stock volatility $\sigma(\cdot, \hat{\pi}^L, \hat{X})$ and its boundaries**

Parameters: $X_0 = 10$ $Y_0 = 100$ $D^* = 1$ $\alpha_y = 0.1$ $\sigma_y = 0.2$

Panel A: $\gamma = 2$ (the large trader's relative risk aversion)



Panel B: $\gamma = 1$ (the large trader's relative risk aversion)



5 Conclusion

In this paper we analyzed the impact of a large trader's optimal investment policy on the stock price process in an economy with finite price elasticity of market demand. The latter property is mainly due to the extrapolative (instead of rational) *expectation* formation proposed by Frey and Stremme (1997). The finiteness of the price elasticity induces a feedback effect on drift and volatility of the stock.

We have shown that the drift and the volatility are higher than in the reference economy where no large trader is in the market. Furthermore, the risk premium remains unchanged and the large investor's optimal final wealth is exactly the same as in an economy where his action has no price impact. Hence, the large trader has neither an advantage nor an disadvantage from the fact that his trades affect the stock price. Obviously, small investors do not have any disadvantage from the presence of the large trader since the stock's risk premium is independent of the large trader's impact on the stock prices. But on the other hand, if the large trader is in the market, drift and volatility of the stock become stochastic and as a consequence, from a small trader's view, the market appears to be incomplete so that he is not able to hedge options written on this stock.

A Replication of Contingent Claims

In this appendix we give a short overview of the theory of replicating a contingent claim in an economy where the stock price depends on the large investor's investment.

For some technical reasons, we have to enlarge the set of wealth processes by also admitting consumption. A (cumulative) *consumption process* is a nonnegative, nondecreasing process c with $c_0 = 0$ and $c_T < \infty$. The wealth process $X^{x,\pi,c}$ corresponding to a trading strategy π and a (cumulative) consumption process c then is the solution of the stochastic differential equation

$$dX_t = X_t\pi_t\alpha_t(\pi_t, X_t)dt + \pi_t X_t\sigma_t(\pi_t, X_t)dW_t - dc_t,\ 0 \le t \le T,\ X_0 = x.$$

We call a pair (π, c) of a trading strategy and a consumption process *admissible* for the initial wealth $x > 0$ if $X_t^{x,\pi,c} \ge 0$ holds for all $t \in [0, T]$ almost surely. The set of admissible pairs of trading strategies and consumption processes is denoted by $A_c(x)$.

Note that the theory presented in this appendix represents the large trader's view. Since the stock price is influenced by the large trader's policy, the market is incomplete from the view of a small investor (who does not know this strategy). So a small investor is *not* able to replicate a contingent claim. Since we only are interested in a replication strategy for the large trader, this does not bother us.

Definition A.1

(a) A *contingent claim* is any F_T- measurable, nonnegative random variable B with $EB^2 < \infty$.

(b) The *arbitrage-free price* of a contingent claim B is defined as

$$h(0) := inf\{x > 0 : \exists(\pi, c) \in A_c(x) \text{ with } X_T^{x,\pi,c} \ge B\}$$

(c) A *replication process* is any wealth process X with $X_T \ge B$ and $X_0 = h(0)$.

(d) A contingent claim B is called *attainable*, if an admissible portfolio $\pi \in A(x)$ exists with $X_T^{x,\pi} = B$ a.s.

For a contingent claim B we introduce a stochastic value process V by setting

$$V(t) := \operatorname*{ess\,sup}_{\lambda \in \mathcal{N}} E^{\lambda} \left[B\beta_{\lambda}(t,T) - \int_t^T \beta_{\lambda}(t,s)\tilde{g}(s,\lambda_s)ds \middle| F_t \right] \tag{37}$$

We now give some lemmata which will be needed for the proof of Proposition A.1.

Lemma A.1

For any contingent claim B the value process $\{V_t\}_{t\in[0,T]}$ satisfies the dynamic programming equation

$$V(t) = \operatorname*{ess\,sup}_{\lambda \in \mathcal{N}} E^{\lambda} \left[V(\theta)\beta_{\lambda}(t,\theta) - \int_t^{\theta} \beta_{\lambda}(t,s)\tilde{g}(s,\lambda_s)ds \middle| F_t \right]$$

for all $\theta \in [t,T]$.

PROOF: The proof is analogous to the proof of proposition 6.2 in Cvitanic (1997a) if the random variable $J_{\lambda}(\theta)$ is defined as

$$J_{\lambda}(\theta) := E^{\lambda} \left[V_T\beta_{\lambda}(\theta,T) - \int_{\theta}^T \beta_{\lambda}(\theta,s)\tilde{g}(s,\lambda_s)ds \middle| F_{\theta} \right]$$

for $t \in [0,T]$ and $\theta \in [t,T]$. □

Lemma A.2

Let us consider the process V in its cadlag modification. Then for every $\lambda \in \mathcal{N}$ the stochastic process Q_{λ} defined by

$$Q_{\lambda}(t) := V(t)\beta_{\lambda}(t) - \int_0^t \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds, \quad 0 \le t \le T,$$

is a $P^{\lambda}-$ supermartingale with cadlag paths.

PROOF: The proof of lemma 6.3 in Cvitanic (1997a) for the economy without a non-price-taking-agent can be repeated using the modified definition of Q_{λ} and V. □

We are now able to proof the following Proposition.

Proposition A.1

For any contingent claim B we have $h(0) = V_0$. Furthermore, there exists a pair $(\hat{\pi}, \hat{c}) \in A_c(x)$ with $X^{V_0,\hat{\pi},\hat{c}} = V$.

PROOF: (following the proof of theorem 3.5 in Cvitanic (1997b))

First, we show $V_0 \geq h(0)$. We therefore look for $(\hat{\pi}, \hat{c}) \in A_0(V_0)$ which satisfies $V = X^{V_0,\hat{\pi},\hat{c}}$. Due to Lemma A.2

$$Q_\lambda(t) = V_t \beta_\lambda(t) - \int_0^t \beta_\lambda(s) \tilde{g}(s, \lambda_s) ds, \quad 0 \leq t \leq T$$

is a $P^\lambda-$ supermartingale for any $\lambda \in \mathcal{N}$. So we get from the Doob-Meyer-decomposition and the martingale representation theorem that for any $\lambda \in \mathcal{N}$ there exist processes ψ_λ and A_λ so that

$$Q_\lambda(t) = V_0 + \int_0^t \psi_\lambda(s) dW_\lambda(s) - A_\lambda(t), \quad 0 \leq t \leq T, \tag{38}$$

ψ_λ is quadratic integrable and A_λ is nondecreasing with $A_\lambda(0) = 0$ and $A_\lambda(T) < \infty$ almost surely. Further, for any $\nu \in \mathcal{N}$ we have

$$Q_\nu(t) \quad = \quad \beta_\nu(t)\beta_\nu^{-1}(t) \left[Q_\nu(t) + \int_0^t \beta_\nu(s)\tilde{g}(s, \nu_s) ds \right] - \int_0^t \beta_\nu(s)\tilde{g}(s, \nu_s) ds.$$

So by Ito's lemma we get

$$\begin{aligned} dQ_\nu(t) \quad = \quad & \beta_\nu(t)\beta_\lambda^{-1}(t)[\psi_\lambda(t) dW_\nu(t) - \psi_\lambda(t)(\kappa_\lambda(t) - \kappa_\nu(t))dt - dA_\lambda(t)] \\ & + V_t\beta_\nu(t)[\nu^0(t) - \lambda^0(t)]dt + \beta_\nu(t)[\tilde{g}(t, \lambda_t) - \tilde{g}(t, \nu_t)]dt \end{aligned} \tag{39}$$

Comparing this with the decomposition

$$dQ_\nu(t) = \psi_\nu(t) dW_\nu(t) - dA_\nu(t) \tag{40}$$

it follows that

$$\psi_\nu(t) = \beta_\lambda(t)\beta_\nu^{-1}(t)\psi_\lambda(t).$$

That is the process $\psi_\lambda \beta_\lambda^{-1}$ is independent of $\lambda \in \mathcal{N}$ and since we have assumed that $\pi \mapsto \pi\sigma(t, \pi, x)$ is invertible for all $t \in [0, T]$ and all $x > 0$, there exists a process $\hat{\pi}$ which satisfies

$$\psi_\lambda(t)\beta_\lambda^{-1}(t) = \hat{\pi}_t V_t \sigma_t(\hat{\pi}_t, V_t), \quad 0 \leq t \leq T.$$

The comparision of (39) and (40) now also yields

$$\begin{aligned}\beta_\nu^{-1}(t)dA_\nu(t) &= \beta_\lambda^{-1}(t)\psi_\lambda(t)V_t(\kappa_\lambda(t)-\kappa_\nu(t))dt+\beta_\lambda^{-1}(t)dA_\lambda(t)\\ &\quad -V_t[\nu^0(t)-\lambda^0(t)]dt-[\tilde g(t,\lambda_t)-\tilde g(t,\nu_t)]dt\end{aligned}$$

or

$$\begin{aligned}&\beta_\nu^{-1}(t)dA_\nu(t)+\hat\pi_tV_t\sigma_t(\hat\pi_tV_t)\kappa_\nu(t)dt+V_t\nu^0(t)dt-\tilde g(t,\nu_t)dt\\ &\quad=\beta_\lambda^{-1}(t)dA_\lambda(t)+\hat\pi_tV_t\sigma_t(\hat\pi_t,V_t)\kappa_\lambda(t)dt+V_t\lambda^0(t)dt-\tilde g(t,\lambda_t)dt\end{aligned}$$

so that the process $\hat c$ defined by

$$\begin{aligned}\hat c_t &:= \int_0^t [V_s\lambda_0(s)-\tilde g(s,\lambda_s)+\hat\pi_sV_s\sigma_s(\hat\pi_s,V_s)\kappa_\lambda(s)+\hat\pi_sV_s\alpha_s)]ds\\ &\quad+\int_0^t\beta_\lambda^{-1}(s)dA_\lambda(s),\quad 0\le t\le T \end{aligned}\tag{41}$$

is also independent of $\lambda\in\mathcal N$. Since there exists a process $\widehat\lambda\in\mathcal N$ with

$$\tilde g(t,\widehat\lambda_t)-\hat\pi_tV_t\alpha_t-V_t\widehat\lambda_0(t)-\hat\pi_tV_t\sigma_t\kappa_{\widehat\lambda}(t)=0,\quad 0\le t\le T, a.s.$$

equation (41) with $\widehat\lambda$ yields $\hat c_t=\int_0^t\beta_{\widehat\lambda}^{-1}(s)dA_{\widehat\lambda}(s)$. So $\hat c$ is nondecreasing, adapted, cadlag with $\hat c_0=0$ and $\hat c_T<\infty$ almost surely. That is $\hat c$ is a consumption process. Furthermore we have

$$\begin{aligned}&d\left(\beta_\lambda(t)V_t-\int_0^t\beta_\lambda(s)\tilde g(s,\lambda_s)ds\right)\\ &\qquad= dQ_\lambda(t)=\psi_\lambda(t)dW_\lambda(t)-dA_\lambda(t)\\ &\qquad= \beta_\lambda(t)\hat\pi_tV_t\sigma_tdW_t-\beta_\lambda(t)\hat c_t+\beta_\lambda(t)\lambda^0(t)V_tdt\\ &\qquad\quad+\beta_\lambda(t)\hat\pi_tV_t\alpha_tdt-\beta_\lambda(t)\tilde g(t,\lambda_t)dt\end{aligned}$$

that is $\beta_\lambda V$ and $\beta_\lambda X^{V_0,\hat\pi,\hat c}$ satisfy the same stochastic differential equation with the same initial values. So it follows $V=X^{V_0,\hat\pi,\hat c}$ and $h(0)\le V_0$ holds.

We now show the inequality $h(0)\ge V_0$ and w.l.o.g. assume $h(0)<\infty$. Then there exists a $x\in(0,\infty)$ and some $(\pi,c)\in A_0(x)$ with $X_T^{\pi,c}\ge B$. For $\lambda\in\mathcal N$ the process

$$\begin{aligned}M_t &:= \beta_\lambda(t)X_t+\int_0^t\beta_\lambda(s)dc_s-\int_0^t\beta_\lambda(s)X_s\left[\pi_s\alpha_t+\lambda_0(s)+\pi_s\sigma_s\kappa_\lambda(s)\right]ds\\ &= x+\int_0^t\beta_\lambda(s)\pi_sX_s\sigma_sdW_\lambda(s)\end{aligned}$$

is a $P^{\lambda}-$ supermartingale. So we get from the inequality

$$\begin{aligned} x &\geq E^{\lambda}[M_T] \geq E^{\lambda}\left[\beta_{\lambda}(T)X_T - \int_0^t \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds\right] \\ &\geq E^{\lambda}\left[\beta_{\lambda}(T)B - \int_0^t \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds\right] \end{aligned}$$

that $V_0 \leq x$ and consequently $V_0 \leq h(0)$. □

We even get the stronger result

Proposition A.2

Any contingent claim B is attainable because the process $\hat{c}$ of Proposition A.1 is identically zero.

PROOF: The proof of this lemma follows that of Theorem 3.7 in Cvitanic (1997b). □

The following Proposition summarizes some equivalent statements.

Proposition A.3

(a) For any contingent claim B with $V_0 < \infty$ and any $\lambda \in \mathcal{N}$ the following statements are equivalent:

(i) $Q_{\lambda}(t) = V(t)\beta_{\lambda}(t) - \int_0^t \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds$ *is a* $P^{\lambda}-$ *martingale.*

(ii) $V_0 = E^{\lambda}\left[B\beta_{\lambda}(T) - \int_0^T \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds\right]$

(iii) B is attainable by a trading strategy π and

$$X_t^{V_0,\pi}\beta_{\lambda}(T) - \int_0^t \beta_{\lambda}(s)\tilde{g}(s,\lambda_s)ds$$

is a $P^{\lambda}-$ martingale.

(b) Each of the statements (i) - (iii) of (a) implies $\hat{c} \equiv 0$ and

$$\tilde{g}_t(\lambda_t) - X_t\lambda^0(t) - \hat{\pi}_t X_t \sigma_t \kappa_{\lambda}(t) - \hat{\pi}_t X_t \alpha_t = 0 \quad a.s. \tag{42}$$

with $(\hat{\pi},\hat{c})$ of Proposition A.1.

Proof: Repeat the proof of Theory 6.6 of Cvitanic (1997a). □

B Proofs of Propositions

Proof of Proposition 1: We first proof the assertion (a). Due to the definition of $\tilde{U}$ we have

$$\begin{aligned} U(X_T) &\leq \tilde{U}(yH_\lambda(T)) + yH_\lambda(T)X_T \qquad (43) \\ &= \tilde{U}(yH_\lambda(T)) + yx \\ &\quad + y\int_0^T H_\lambda(t)X_t[\pi_t\alpha_t - \lambda_0(t) - \kappa_\lambda(t)\pi_t\sigma_t]dt \\ &\quad + y\int_0^T H_\lambda(t)X_t[\pi_t\sigma_t - \kappa_\lambda(t)]dW_t \\ &\leq \tilde{U}(yH_\lambda(T)) + yx + y\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt \\ &\quad + y\int_0^T H_\lambda(t)X_t[\pi_t\sigma_t + \kappa_\lambda(t)]dW_t. \qquad (44) \end{aligned}$$

So we get

$$E(U(X_T)) \leq E\left[\tilde{U}(yH_\lambda(T)) + y\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right] + yx \qquad (45)$$

The assertion of (b) is obvious. For the proof of (c) note that equality holds in (45) if and only if equality holds in (43) and (44). Since equality in (43) is equivalent to

$$X_T = I(yH_\lambda(T))$$

and equality in (44) is equivalent to equality in equation (34), the statement (c) follows. □

To proof Proposition 2 we need the following Lemma.

Lemma B.1

Let $y > 0$ and $\widehat{\lambda} = \widehat{\lambda}_y$ be a solution of the dual optimization problem (31). If

$$E\left[H_{\widehat{\lambda}}(T)I(yH_\lambda(T)) + \int_0^T H_\lambda(s)\tilde{g}(s,\lambda(s))ds\right] < \infty$$

holds for all $\lambda \in \mathcal{N}$, then we have for $B = I(yH_{\widehat{\lambda}}(T))$

$$E^\lambda\left[\beta_\lambda(T)B - \int_0^T \beta_\lambda(s)\tilde{g}(s,\lambda_s)ds\right] \;\leq\; E^{\widehat{\lambda}}\left[\beta_{\widehat{\lambda}}(T)B - \int_0^T \beta_{\widehat{\lambda}}(s)\tilde{g}(s,\widehat{\lambda}_s)ds\right].$$

Proof: (following the proof of Lemma 5.12 in Cvitanic (1997b))

For $\epsilon \in (0,1)$ and $\lambda \in \mathcal{N}$ define processes

$$\begin{aligned} G_\epsilon &:= (1-\epsilon)H_{\widehat{\lambda}} + \epsilon H_\lambda \\ \lambda^\epsilon &:= ((1-\epsilon)H_{\widehat{\lambda}}\widehat{\lambda} + \epsilon H_\lambda \lambda)/G_\epsilon \end{aligned}$$

where $\lambda^\epsilon \in \mathcal{N}$ since $\mathcal{N}$ is convex. We then have

$$dG_\epsilon(t) = G_\epsilon(t)\lambda^\epsilon(t)dt + G_\epsilon(t)\kappa_{\lambda^\epsilon}(t)dW_t$$

and since $G_\epsilon(0) = 1 = H_{\lambda^\epsilon}(0)$ we get $G_\epsilon = H_{\lambda^\epsilon}$ a.s.

From the optimality of $\widehat{\lambda}$ it follows

$$E\left[\tilde{U}(yH_{\widehat{\lambda}}(T)) + y\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt\right] \le E\left[\tilde{U}(yG_\epsilon(T)) + y\int_0^T G_\epsilon(t)\tilde{g}(t,\lambda_t^\epsilon)dt\right]$$

or

$$\frac{1}{\epsilon}EV_\epsilon \le 0$$

where we define the random variable V_ϵ as

$$V_\epsilon := \tilde{U}(yH_{\widehat{\lambda}}(T)) - \tilde{U}(yG_\epsilon(T)) + y\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt - y\int_0^T G_\epsilon(t)\tilde{g}(t,\lambda_t^\epsilon)dt.$$

Using the inequality $U(I(y)) \ge U(x) + y[I(y) - x]$, which holds for any $x > 0$ and for any $y > 0$, we derive

$$\begin{aligned} \frac{1}{\epsilon}[\tilde{U}(yH_{\widehat{\lambda}}(T)) - \tilde{U}(yG_\epsilon(T))] &\ge \frac{1}{\epsilon}yI(yG_\epsilon(T))[G_\epsilon(T) - H_{\widehat{\lambda}}(T)] \\ &= yI(yG_\epsilon(T))[H_\lambda(T) - H_{\widehat{\lambda}}(T)] \\ &\overset{\epsilon\to 0}{\longrightarrow} I(yH_{\widehat{\lambda}})[H_\lambda(T) - H_{\widehat{\lambda}}(T)]. \end{aligned} \tag{46}$$

Further, it follows from the convexity of $\tilde{g}$ that

$$\begin{aligned} &\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt - \int_0^T G_\epsilon(t)\tilde{g}(t,\lambda_t^\epsilon)dt \\ &\quad\ge \int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt - (1-\epsilon)\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt \\ &\qquad -\epsilon\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt \\ &\quad= \epsilon\left[\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt - \int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right]. \end{aligned} \tag{47}$$

The inequalities (46) and (47) yield

$$\begin{aligned}\liminf_{\epsilon\to 0}\frac{1}{\epsilon}V_\epsilon \quad &\geq I(yH_{\widehat{\lambda}}(T))[H_\lambda(T)-H_{\widehat{\lambda}}(T)]\\ &+y\left[\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt-\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right].\end{aligned}$$

Application of Fatou's lemma provides the upper bound

$$\begin{aligned}&E\left(I(yH_{\widehat{\lambda}}(T))[H_{\widehat{\lambda}}(T)-H_{\widehat{\lambda}}(T)]+y\int_0^T H_{\widehat{\lambda}}(t)\tilde{g}(t,\widehat{\lambda}_t)dt-y\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right)\\ &\qquad\leq\ E\liminf_{\epsilon\to 0}\frac{1}{\epsilon}V_\epsilon\\ &\qquad\leq\ \liminf_{\epsilon\to 0}\frac{1}{\epsilon}EV_\epsilon\\ &\qquad\leq\ 0. \qquad (48)\end{aligned}$$

The application of Fatou's lemma is justified by an argument of Cvitanic (1997b, p. 242). By recognizing that

$$E\left[\int_0^T H_\lambda(t)\tilde{g}(t,\lambda_t)dt\right]=E\left[Z_\lambda(T)\int_0^T \beta_\lambda(t)\tilde{g}(t,\lambda_t)dt\right]$$

the assertion follows from inequality (48). □

Proof of Proposition 2: Due to Proposition A.1 and Lemma B.1 we have

$$\begin{aligned}h_y(0)=V_0 \quad &= \quad \sup_{\lambda\in\mathcal{N}}E^\lambda\left[B\beta_\lambda(T)-\int_0^T\beta_\lambda(s)\tilde{g}(s,\lambda_s)ds\right]\\ &= \quad E\left[BH_{\widehat{\lambda}}(T)-Z_\lambda(T)\int_0^T\beta_{\widehat{\lambda}}(s)\tilde{g}(s,\widehat{\lambda}_s)ds\right].\end{aligned}$$

Furthermore, due to Proposition A.2 and Proposition A.3 the triplet $(\hat{\pi},\widehat{\lambda},\hat{c})$ in the proof of Proposition A.1 satisfies equation (34) and $\hat{c}\equiv 0$. Since $X_0=V_0$ and $X_T^{\hat{\pi}}=V_T=B$ the portfolio process $\hat{\pi}$ is a replication strategy for the claim B with initial wealth $h_y(0)$. □

Proof of Proposition 3: First we look for the process $\psi_{\widehat{\lambda}}$ which determines the optimal portfolio process $\hat{\pi}$. Due to Proposition A.1 the process $\psi_{\widehat{\lambda}}$ must satisfy

$$dQ_{\widehat{\lambda}}(t)=\psi_{\widehat{\lambda}}(t)dW_{\widehat{\lambda}}(t), \quad 0\leq t\leq T,$$

where

$$Q_{\widehat{\lambda}}(t)=X_t\beta_{\widehat{\lambda}}(t)=E^{\widehat{\lambda}}[I(yH_{\widehat{\lambda}}(T))\beta_{\widehat{\lambda}}(T)|F_t].$$

From equation (36) it follows

$$\begin{aligned} Q_{\hat\lambda}(t) &= E^{\hat\lambda}[(yH_{\hat\lambda}(T))^{-\frac{1}{\gamma}}\beta_{\hat\lambda}(T)|F_t] \\ &= y^{-\frac{1}{\gamma}}(\beta_{\hat\lambda}(T))^{1-\frac{1}{\gamma}}E^{\hat\lambda}[(Z_{\hat\lambda}(T))^{-\frac{1}{\gamma}}|F_t] \\ &= y^{-\frac{1}{\gamma}}(\beta_{\hat\lambda}(T))^{1-\frac{1}{\gamma}}\frac{1}{Z_{\hat\lambda}(t)}E[(Z_{\hat\lambda}(T))^{1-\frac{1}{\gamma}}|F_t]. \end{aligned}$$

Since $Z_{\hat\lambda}$ is a martingale it follows

$$\begin{aligned} & E[(Z_{\hat\lambda}(T))^{1-\frac{1}{\gamma}}|F_t] \\ &= E\left[\exp\left\{-(1-\frac{1}{\gamma})\int_0^T \kappa_{\hat\lambda}(s)dW_s - \frac{1}{2}(1-\frac{1}{\gamma})\int_0^T (\kappa_{\hat\lambda}(s))^2 ds\right\}\Big|F_t\right] \\ &= \exp\left\{-(1-\frac{1}{\gamma})\int_0^t \kappa_{\hat\lambda}(s)dW_s - \frac{1}{2}\int_0^t ((1-\frac{1}{\gamma})\kappa_{\hat\lambda}(s))^2 ds - \frac{1}{2}\frac{1}{\gamma}(1-\frac{1}{\gamma})\int_0^T (\kappa_{\hat\lambda}(s))^2 ds\right\}. \end{aligned}$$

So we get

$$Q_{\hat\lambda}(t) = L_T \exp\left\{\frac{1}{\gamma}\int_0^t \kappa_{\hat\lambda}(s)dW_s - \frac{1}{2}\frac{1}{\gamma^2}\int_0^t (\kappa_{\hat\lambda}(s))^2 ds\right\}$$

with a random variable L_T and it follows

$$dQ_{\hat\lambda}(t) = \frac{1}{\gamma}\kappa_{\hat\lambda}(t)Q_{\hat\lambda}(t)dW_{\hat\lambda}(t)$$

and hence the process $\psi_{\hat\lambda}$ is given by

$$\psi_{\hat\lambda}(t) = \frac{1}{\gamma}\kappa_{\hat\lambda}(t)\beta_{\hat\lambda}(t)X_t, \quad 0 \le t \le T.$$

To determine the optimal portfolio process $\hat\pi$ we solve the equation

$$\hat\pi_t\sigma(t,\hat\pi_t,X_t) = \frac{1}{\gamma}\kappa_{\hat\lambda}(t). \tag{49}$$

Recalling the definition of $\sigma(t,\pi,x)$ in equation (16), equation (49) is equivalent to

$$\hat\pi_t\frac{D^*Y_t}{D^*Y_t+\hat\pi_t(1-\hat\pi_t)X_t}\sigma_y = \frac{1}{\gamma}\kappa_{\hat\lambda}(t) = \frac{1}{\gamma}\frac{\alpha_y}{\sigma_y}$$

which is equivalent to

$$\hat\pi_t^2 - \hat\pi_t\left(1-\gamma\frac{\sigma_y^2}{\alpha_y}\frac{Y_t}{X_t}D^*\right) - \frac{Y_t}{X_t}D^* = 0 \tag{50}$$

and we derive the two possible portfolio processes given in the Proposition. □

C Notation

Y	reference investor's wealth process with $dY_t = Y_t\alpha_y dt + Y_t\sigma_y dW_t$
D^*	reference trader's stock demand if his wealth and the stock price are 1
u	reference trader's utility function
X	large investor's wealth process with $dX_t = X_t\pi_t\alpha(t,\pi_t,X_t)dt + X_t\pi_t\sigma(t,\pi_t,X_t)dt$
π	proportion of large investor's wealth invested in the stock
U	large investor's utility function
I	inverse of U'
S	stock price process with $dS_t = S_t\alpha(t,\pi_t,X_t)dt + S_t\sigma(t,\pi_t,X_t)dt$
W	standard Brownian motion
W_λ	standard Brownian motion w.r.t P^λ : $W_\lambda(t) = W_t + \int_0^t \kappa_\lambda(s)ds$
B	contingent claim
$h(0)$	arbitrage-free price of a contingent claim
$\tilde{g}(t,\lambda_t)$	convex conjugate with $\tilde{g}(t,\lambda_t) = \sup_{\pi,x}[\pi x\alpha_t + x(1-\pi\sigma_t)\cdot\lambda^0(t) + \pi x\sigma_t\cdot\lambda^1(t)]$
$\mathcal{N}_t$	effective domain with $\mathcal{N}_t = \{\lambda \in I\!R^2 : \tilde{g}(t,\lambda_t) < \infty\}$
$\mathcal{N}$	$= \{\lambda : \lambda_t \in \mathcal{N}_t,\ 0 \le t \le T\}$
$\kappa_\lambda(t)$	$= \frac{\overline{\alpha}_t}{\overline{\sigma}_t} + \lambda^1(t)$
Z	exponential martingale with $Z_\lambda(t) = \exp\{-\int_0^t \kappa_\lambda(s)dW_s - \frac{1}{2}\int_0^t \kappa_\lambda^2(s)ds\}$
β_λ	discount factor with $\beta_\lambda(u,t) = \exp\{\int_u^t \lambda^0(s)ds\}$ and $\beta_\lambda(t) = \beta_\lambda(0,t)$
H	stochastic deflator process with $H_\lambda(t) = Z_\lambda(t)\beta_\lambda(t)$
V_t	$= \operatorname{ess\,sup}_{\lambda\in\mathcal{N}} E^\lambda\left[B\beta_\lambda(t,T) - \int_t^T \beta_\lambda(t,s)\tilde{g}(s,\lambda_s)ds\right]$
$Q_\lambda(t)$	$= V_t\beta_\lambda(t) - \int_0^t \beta_\lambda(s)\tilde{g}(s,\lambda(s))ds$

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